Gravitational Many-Body Problem

Jun Makino

Center for Computational Astrophysics
and
Division Theoretical Astronomy
National Astronomical Observator of Japan
Talk structure

1. Overview of Gravitational Many-Body Problem
   - collisionless and collisional systems
   - Evolution of collisional systems

2. Some resent topics
   - Black hole formation in star clusters

3. Simulation methods
Gravitational Many-Body Problem

- Definition
- Astronomical examples
- Large-$N$ limit — “collisionless” systems
- Two-body (collisional) relaxation
- Evolution of collisional systems
Definition

\[ m_i \frac{d^2 x_i}{dt^2} = \sum_{j \neq i} f_{ij} \]  \hspace{1cm} (1)

\( x_i, m_i \): position and mass of particle \( i \)

\( f_{ij} \): force from particle \( j \) to particle \( i \)

Gravitational (Newtonian) force:

\[ f_{ij} = G m_i m_j \frac{x_j - x_i}{|x_j - x_i|^3} \]  \hspace{1cm} (2)

\( G \): Gravitational constant
Astronomical Examples

Star Clusters

Galaxy

M3 (the Great Globular Cluster in Hercules)

M51 (the Whirlpool Galaxy)
Galaxy Group
Clusters of Galaxies

Galaxy Cluster 0024+1645

Yellow: Galaxies in the cluster

Blue: background galaxies (enlarged and stretched through gravitational lens effect)
Large Scale Structure


SDSS Survey
Radius: about 1Gpc (1/3 of the Universe)
Constituent of the Universe

70%: Interacts with nothing
25%: Interacts only through gravity
Behavior of Dark Matter

Initial conditions

- Power spectrum of density fluctuation
- Cosmological parameters

Fairly well understood (if our assumption on the nature of DM is correct)
We can simulate the structure formation through dark matter dynamics
(Animation)
Large-$N$ Limit
— collisionless systems

Collisionless Boltzmann Equation (CBE)

\[
\frac{\partial f}{\partial t} + v \cdot \nabla f - \nabla \Phi \cdot \frac{\partial f}{\partial v} = 0,
\]

(3)

$f$: Distribution function in 6-dimensional phase space

$\Phi$: Gravitational potential, given by:

\[
\nabla^2 \Phi = 4\pi G \rho.
\]

(4)

$\rho$: mass density

\[
\rho = m \int dv f,
\]

(5)
Dynamical Equilibrium

Dynamical Equilibrium: Stationary solution of CBE+Poisson Eq.

Star clusters, Galaxies: Roughly in DE
Large Scale Structure: NOT in DE
(Dynamically too young)
Jeans’ Theorem

Stationary solution of CBE in a given potential $\phi$ can be expressed as

$$f(x, v) = f[I_1(x, v), I_2(x, v), ...],$$

where $I_1, I_2, ...$ are integrals of the motion.

Example: Spherically symmetric system:
$f = f(E, J)$. (energy and total angular momentum)

The distribution function $f$ for a stellar system with finite mass cannot be Maxwellian, because $f$ must be zero for $E > 0$. 
Dynamical and Thermal Equilibrium

Dynamical Equilibrium: Stationary state of CBE

No entropy production (no collision term)

→ Not in thermal equilibrium
Not in LOCAL thermal equilibrium ether
(Jeans’ Theorem)
Collision term

Two-body relaxation
Close encounter between two point-mass particles
Orbit changes: similar to physical collision
Relaxation timescale:

\[ t_r \sim \frac{v^3}{Gnm^2 \log \Lambda} \]  \hspace{1cm} (7)

\[ \log \Lambda: \text{ Coulomb Logarithm, } \log \Lambda \sim \log N \]

\[ t_r \sim \frac{N}{\log N} t_d \]  \hspace{1cm} (8)

\[ t_d: \text{ Dynamical timescale} \]
Relaxation timescale \( \propto \text{ Number of particles} \)
Thermal (two-body) relaxation

- Important for
  - Small-$N$ systems
  - Systems with small dynamical timescale

- Examples
  - Star clusters
  - Central region of galaxies
  - Clusters of Galaxies
  - Few-body systems (triple etc)
Virial Theorem and negative specific heat

For a stellar system in dynamical equilibrium

\[ E = -K = \frac{V}{2} \quad (9) \]

\( E \): Total energy (= \( K + V \))
\( K \): Kinetic energy
\( V \): Potential energy

If a system loses energy (\( \Delta E < 0 \)), it becomes hotter (\( \Delta K > 0 \))
Gravothermal Instability

Isothermal gas in a spherical adiabatic wall

Radius: $R$
Mass: $M$
Central density: $\rho_0$
Density at the wall: $\rho_w$

$D = \rho_0/\rho_w$

$D = 1$: No gravity
$D = \infty$: Singular solution

$\rho \propto r^{-2}$
Gravothermal Instability (cont’d)

For large $D$ ($D > 709$):

move heat from the central region to outer Isothermal gas

↓

central region becomes hotter due to negative specific heat

↓

heat flow grows
Evolution in finite amplitude

Numerical calculation
Hachisu et al. (1978): Gas dynamics

Lynden-Bell & Eggleton (1980): Self-similar solution

Cohn (1980): Direct integration of orbit-averaged Fokker-Planck equation
Energy Production

Gravitational contraction of star: halted by nuclear fusion

Stellar system: Three-body binary formation

Close encounter of three particles $\rightarrow$

One binary $+$ one single star:
  Statistically most likely outcome
Animation

Three particles
Gravothermal Oscillation

Sugimoto and Bettwieser (1983)

Three curves:
Different energy production coefficients
(Different $N$)
$N$-body simulation

1996: Confirmed with $N$-body simulation
Some recent topics

- Collision of stars and blackhole formation
IMBH candidate in M82


X-ray sources. Brightest one: Corresponds to 1000-solar-mass black hole (Intermediate-mass BH)
Subaru observation
Host star cluster

McCrady et al. 2003 (astro-ph/0306373)
Cluster #11 (MGG-11)

- $\sigma_r = 11.4 \pm 0.8 \text{km/s}$
- half-light radius $1.2 \pm 0.17 \text{pc}$
- kinetic mass $3.5 \pm 0.7 \times 10^5 M_\odot$
- Age $\sim 10 \text{Myrs}$.

Very short relaxation time (less than 10Myrs)
A possible scenario

1. Massive stars sink to the center through relaxation

2. Supermassive star forms through collision and merging

3. This star collapses to a BH
Simulation

Portegies Zwart et al., Nature, Apr 15, 2004

This scenario seems to work
GRAPE-6
Special-purpose computer for $N$-body simulations.
64 Tflops peak — World’s fastest computer as of 2002.
Basic idea of GRAPE

Special-purpose hardware for force calculation
General-purpose host for all other calculation

\[
\frac{d^2 x_i}{dt^2} = \sum_{j \neq i} -Gm_j \frac{x_i - x_j}{|x_i - x_j|^3}
\]

Host Computer \( x, m \) \( \rightarrow \) GRAPE

Time integration, IO, etc
Flexibility

Force calculation
High performance
Calculates gravitational force, its first time derivative and potential.
Evolution of GRAPE systems
Next-Generation GRAPE — GRAPE-DR

- Planned peak speed: 2 Pflops
- New architecture — wider application range than previous GRAPEs
- primarily to get funded
- No force pipeline. SIMD programmable processor
- Planned completion year: FY 2008 (early 2009)
Processor architecture

- Float Mult
- Float add/sub
- Integer ALU
- 32-word registers
- 256-word memory
- communication port
Collection of small processors.

512 processors on one chip
500MHz clock

Peak speed of one chip: 0.5 Tflops (20 times faster than GRAPE-6).
Development status

Sample chip delivered May 2006
Chip layout

- 32PES in 16 groups
- 18mm by 18mm
Prototype board

2nd prototype board. (Designed by Toshi Fukushige)
Difference from the 1st one:
PCI-Express x8 interface
On-board DRAM
Designed to run real applications
Summary

- Gravitational Many-body problem is fun
- Recently, numerical integration of $N$-body systems have become useful tool for modelling observations.
- Special-purpose computers help.