Gravitational Many-Body Problem

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Talk structure

1. Overview of Gravitational Many-Body Problem
   - collisionless and collisional systems
   - Evolution of collisional systems
2. Some resent topics
   - Black hole formation in star clusters
3. Simulation methods

Gravitational Many-Body Problem

- **Definition**
- **Astronomical examples**
- **Large-N limit — "collisionless" systems**
- **Two-body (collisional) relaxation**
- **Evolution of collisional systems**

Definition

$$m_i \frac{d^2 x_i}{dt^2} = \sum_{j \neq i} f_{ij} \tag{1}$$

x_i, m_i: position and mass of particle i
f_{ij}: force from particle j to particle i
Gravitational (Newtonian) force:

$$f_{ij} = G m_i m_j \frac{x_j - x_i}{|x_j - x_i|^3}, \tag{2}$$

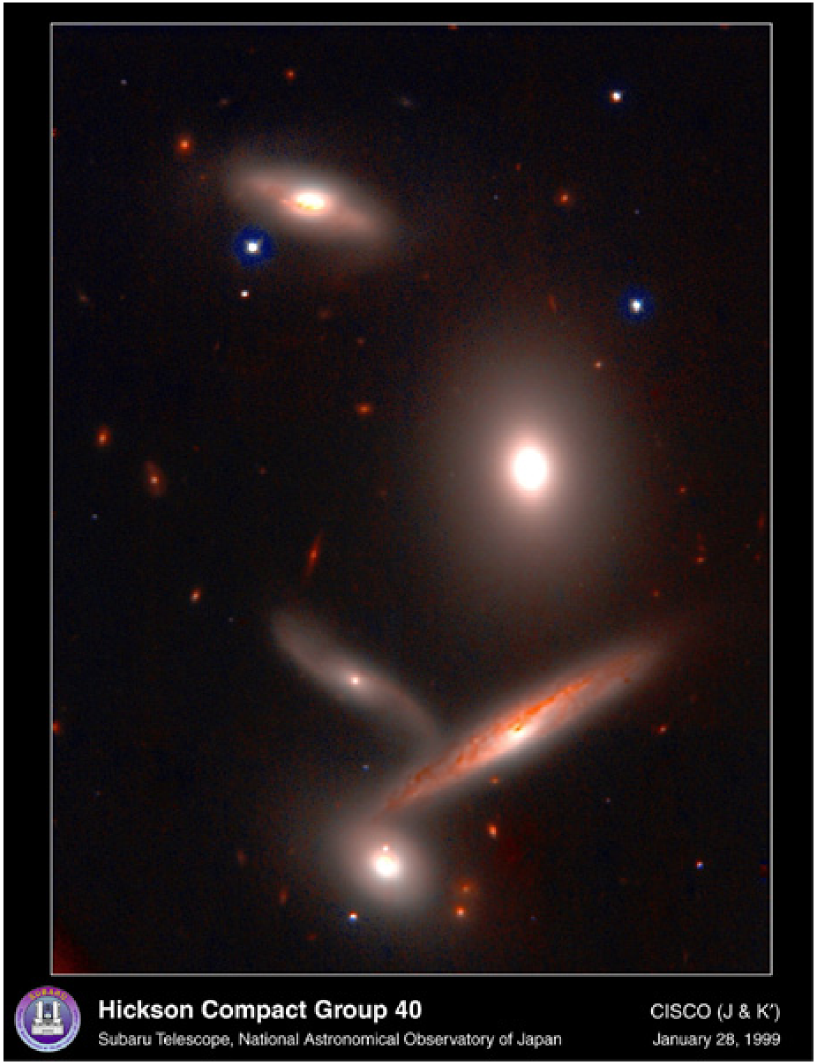
G : Gravitational constant

Astronomical Examples

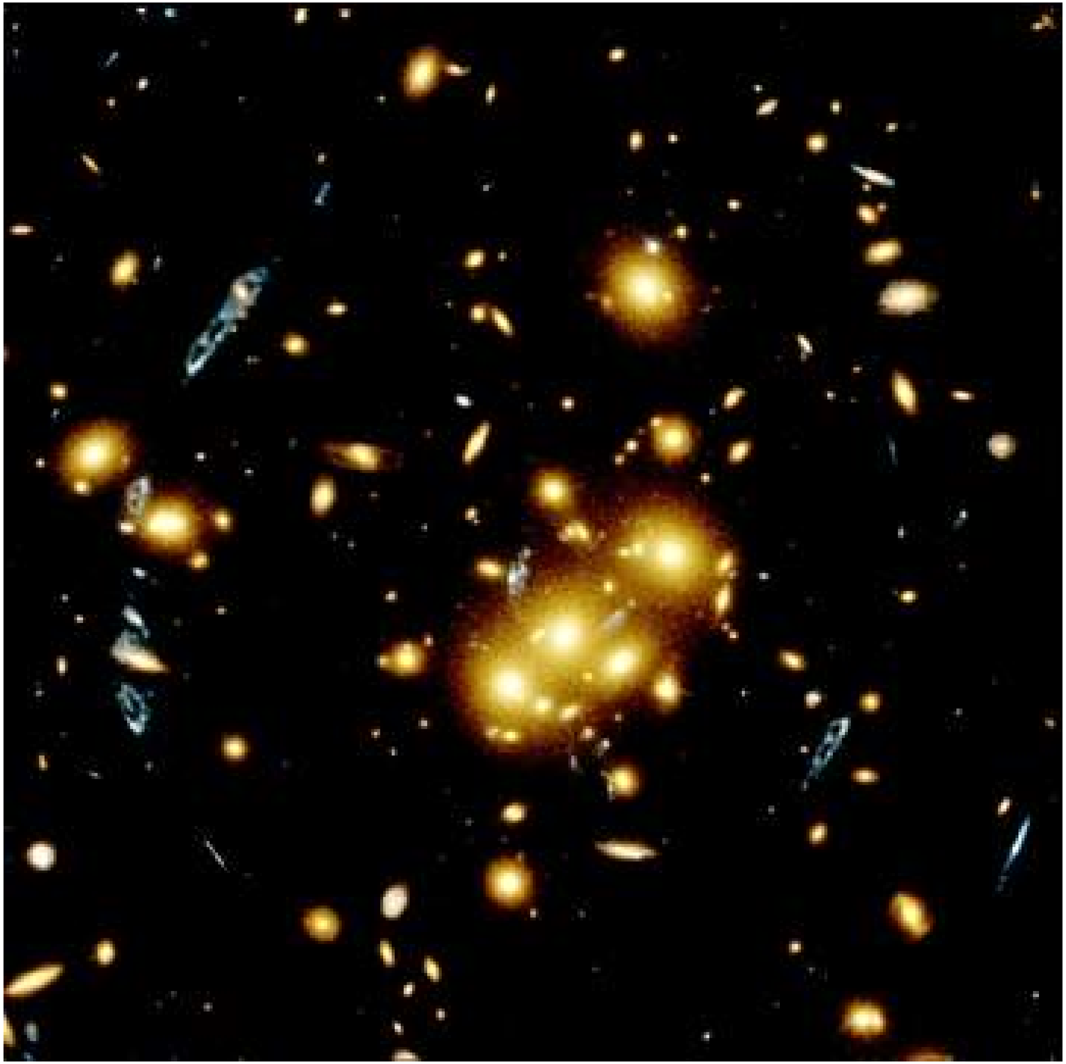
Star Clusters

Galaxy

Galaxy Group



Clusters of Galaxies

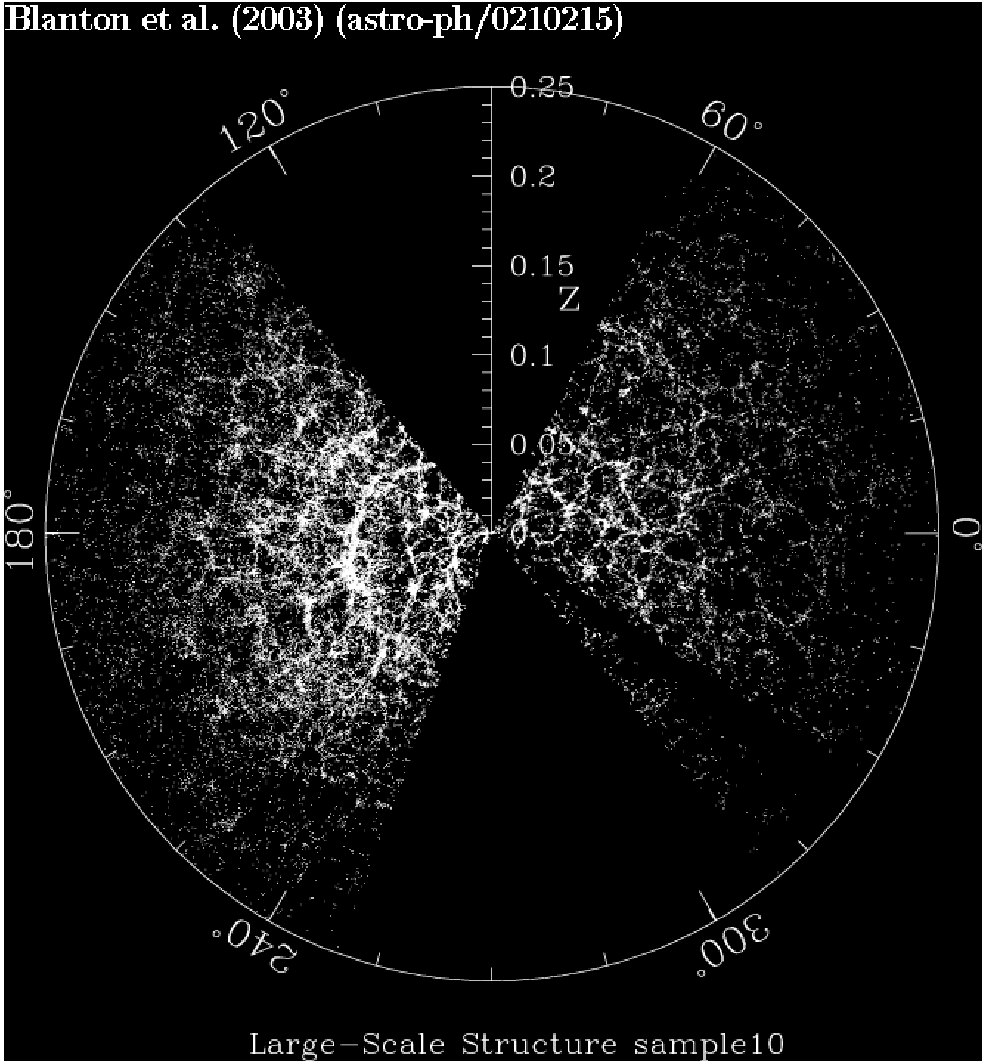


Galaxy Cluster 0024+1645

Yellow: Galaxies in the cluster

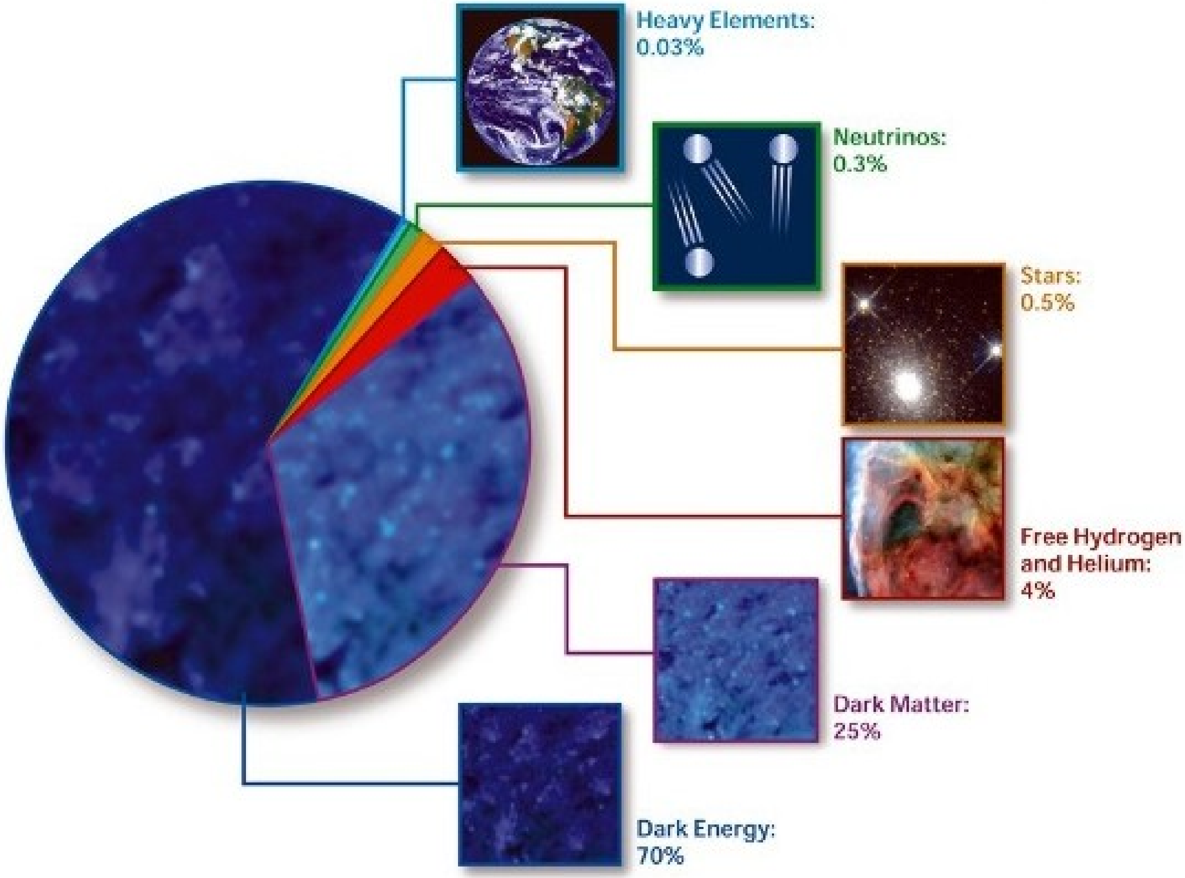
Blue: background galaxies (enlarged and stretched through gravitational lens effect)

Large Scale Structure



SDSS Survey Radius: about 1Gpc (1/3 of the Universe)

Constituent of the Universe



70%: Interacts with nothing
25%: Interacts only through gravity

Behavior of Dark Matter

Initial conditions

- **Power spectrum of density fluctuation**
- **Cosmological parameters**

Fairly well understood (if our assumption on the nature of DM is correct)
We can simulate the structure formation through dark matter dynamics
(Animation)

Large-N Limit — collisionless systems

Collisionless Boltzmann Equation (CBE)

$$\frac{\partial f}{\partial t} + v \cdot \nabla f - \nabla\Phi \cdot \frac{\partial f}{\partial v} = 0, \tag{3}$$

f : Distribution function in 6-dimensional phase space

Φ : Gravitational potential, given by:

$$\nabla^2\Phi = 4\pi G\rho. \tag{4}$$

ρ: mass density

$$\rho = m \int dv f, \tag{5}$$

Dynamical Equilibrium

Dynamical Equilibrium: Stationary solution of CBE+Poisson Eq.

Star clusters, Galaxies: Roughly in DE
Large Scale Structure: NOT in DE
(Dynamically too young)

Jeans' Theorem

Stationary solution of CBE in a given potential ϕ can be expressed as

$$\boldsymbol{f(x, v) = f[I_1(x, v), I_2(x, v), ...]}, \tag{6}$$

where $I_1, I_2, ...$ are integrals of the motion.

Example: Spherically symmetric system:
$f = f(E, J)$. (energy and total angular momentum)

The distribution function f for a stellar system with finite mass cannot be Maxwellian, because f must be zero for $E > 0$.

Dynamical and Thermal Equilibrium

Dynamical Equilibrium: Stationary state of CBE

No entropy production (no collision term)

$\rightarrow$ Not in thermal equilibrium
Not in LOCAL thermal equilibrium ether
(Jeans' Theorem)

Collision term

Two-body relaxation
Close encounter between two point-mass particles
Orbit changes: similar to physical collision
Relaxation timescale:

$$t_r \sim \frac{v^3}{Gnm^2 \log \Lambda} \tag{7}$$

$\log \Lambda$: Coulomb Logarithm, $\log \Lambda \sim \log N$

$$t_r \sim \frac{N}{\log N} t_d \tag{8}$$

t_d: Dynamical timescale
Relaxation timescale $\propto$ Number of particles

Thermal (two-body) relaxation

- **Important for**
  - **Small-N systems**
  - **Systems with small dynamical timescale**
- **Examples**
  - **Star clusters**
  - **Central region of galaxies**
  - **Clusters of Galaxies**
  - **Few-body systems (triple etc)**

Virial Theorem
and negative specific heat

For a stellar system in dynamical equilibrium

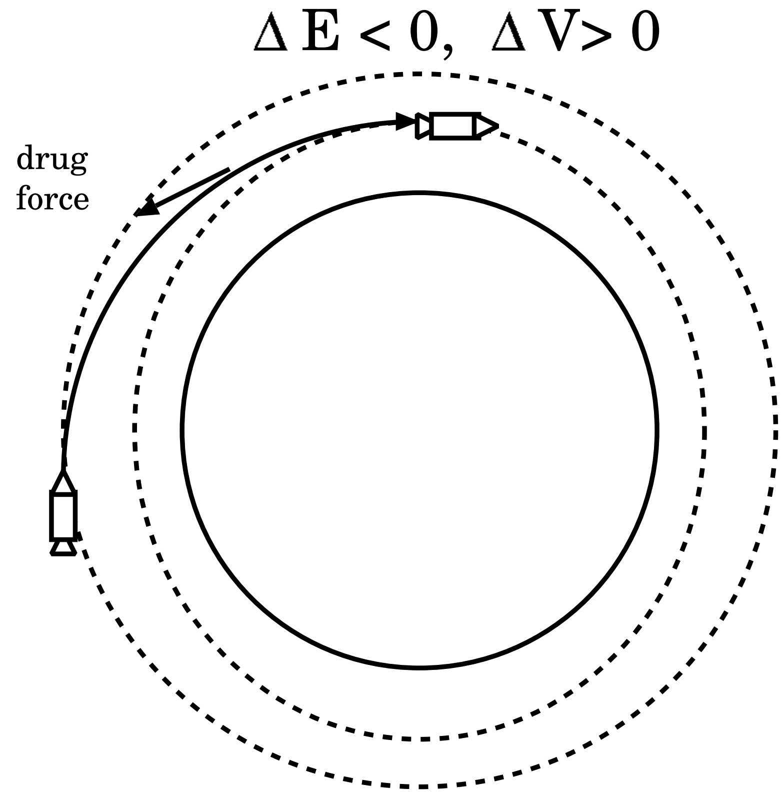
$$\boldsymbol{E = -K = V/2} \qquad (9)$$

$\boldsymbol{E}$: **Total energy (=** $\boldsymbol{K+V}$**)**

$\boldsymbol{K}$: **Kinetic energy**

$\boldsymbol{V}$: **Potential energy**

If a system loses energy ($\boldsymbol{\Delta E < 0}$), it becomes hotter ($\boldsymbol{\Delta K > 0}$)

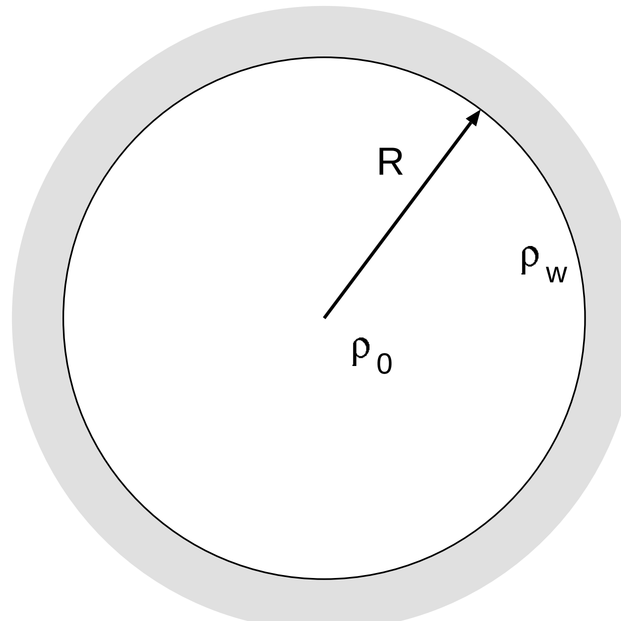


Gravothermal Instability

Isothermal gas in
a spherical adiabatic wall

Radius: $\boldsymbol{R}$
Mass: $\boldsymbol{M}$
Central density: $\boldsymbol{\rho_0}$
Density at the wall: $\boldsymbol{\rho_w}$
$\boldsymbol{D = \rho_0/\rho_w}$

$\boldsymbol{D = 1}$**: No gravity**
$\boldsymbol{D = \infty}$**: Singular solution**
$\boldsymbol{\rho \propto r^{-2}}$



Gravothermal Instability(cont'd)

For large D ($D > 709$):

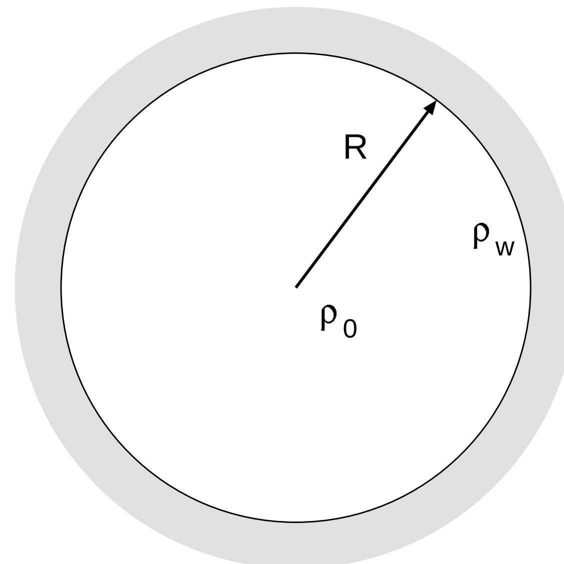
move heat from the central region to outer Isothermal gas

$\downarrow$

central region becomes hotter due to negative specific hear

$\downarrow$

heat flow grows

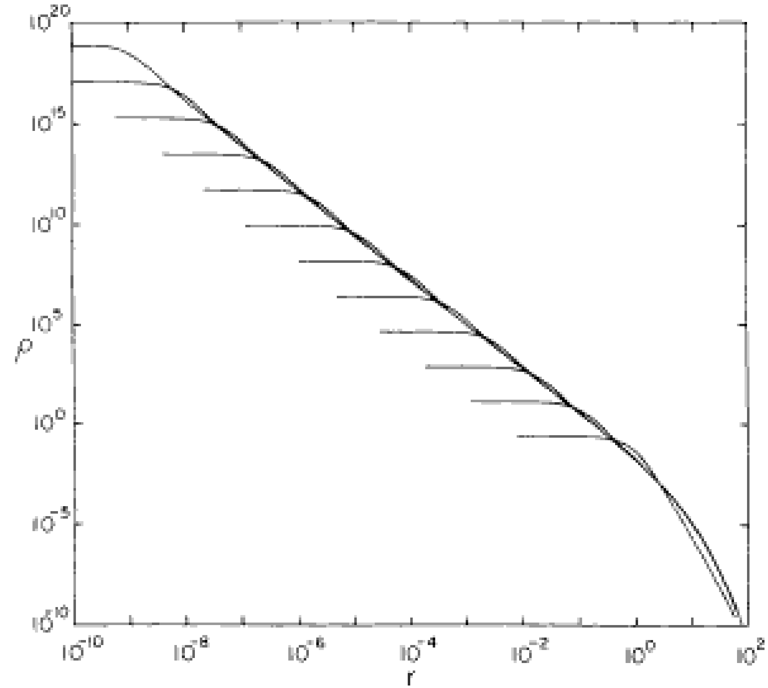


Evolution in finite amplitude

Numerical calculation
Hachisu *et al.* (1978) :
Gas dynamics

Lynden-Bell & Eggleton (1980): Self-similar solution

Cohn (1980): Direct integration of orbit-averaged Fokker-Planck equation



Energy Production

Gravitational contruction of star:
halted by nuclear fusion

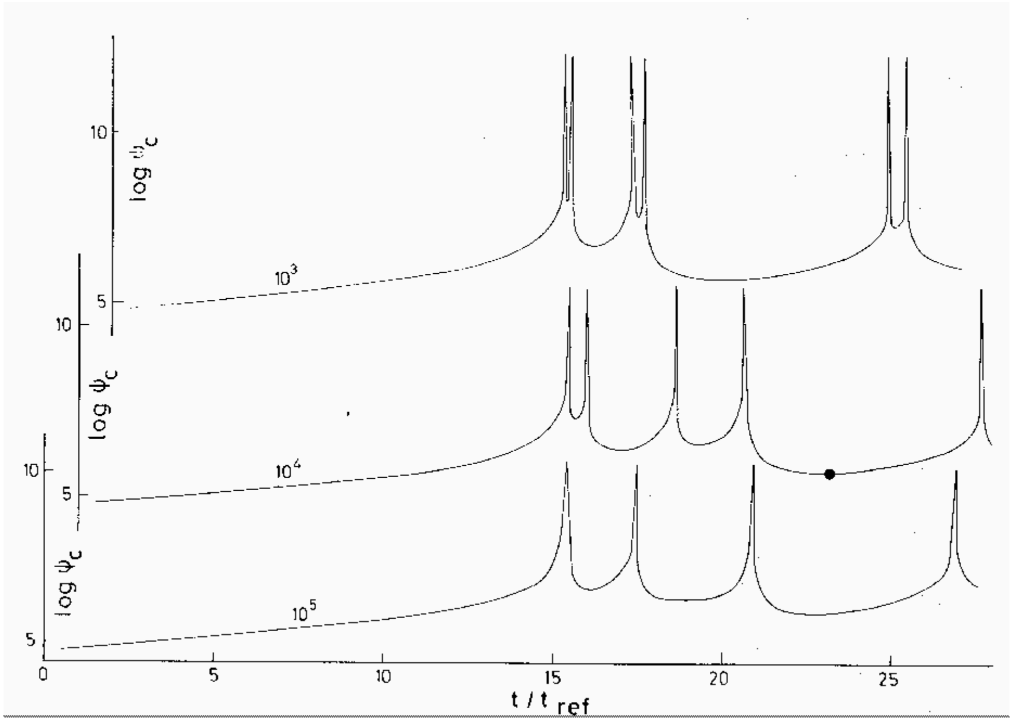
Stellar system: Three-body binary formation

Close encounter of three particles $\rightarrow$
One binary + one single star:
Statistically most likely outcome

Animation

Three particles

Gravothermal Oscillation

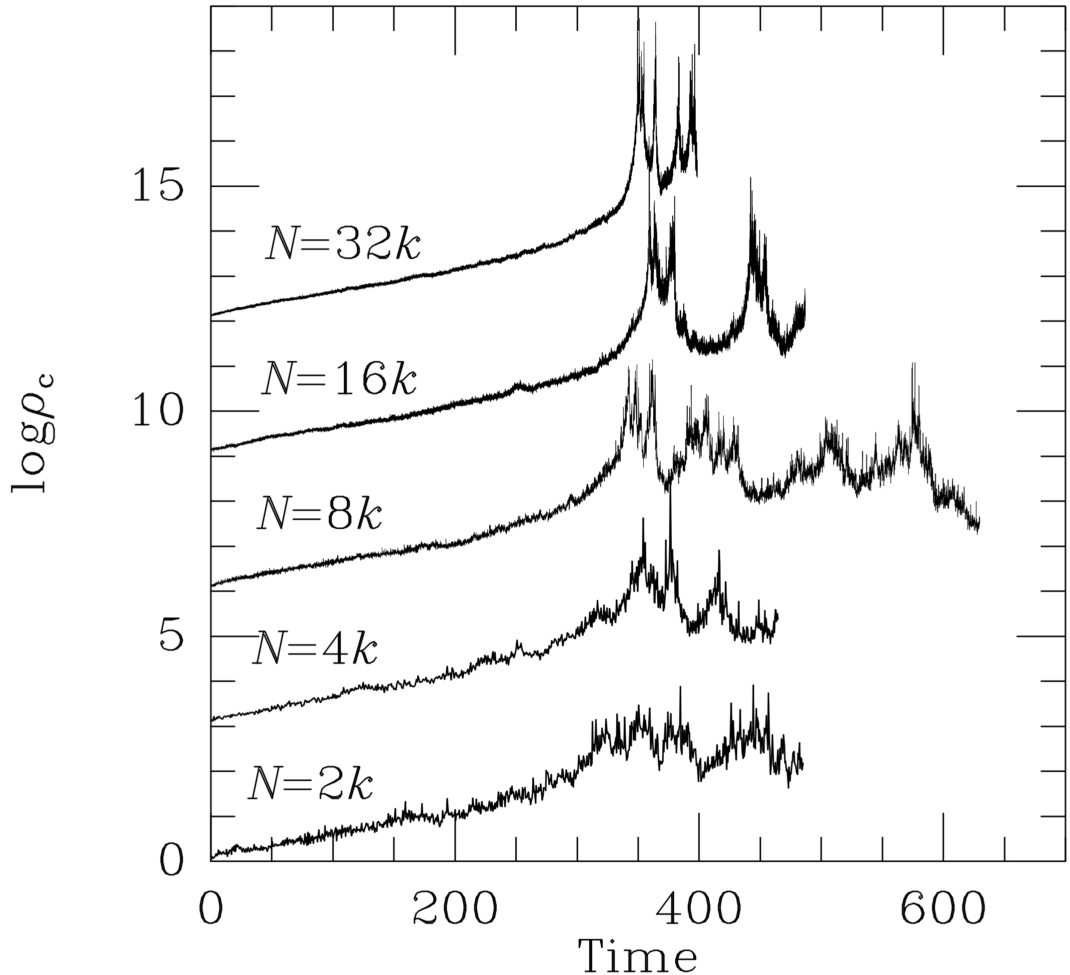


Sugimoto and Bettwieser (1983)

Three curves: Different energy production coefficients (Different N)

N-body simulation

1996 : Confirmed with N-body simulation

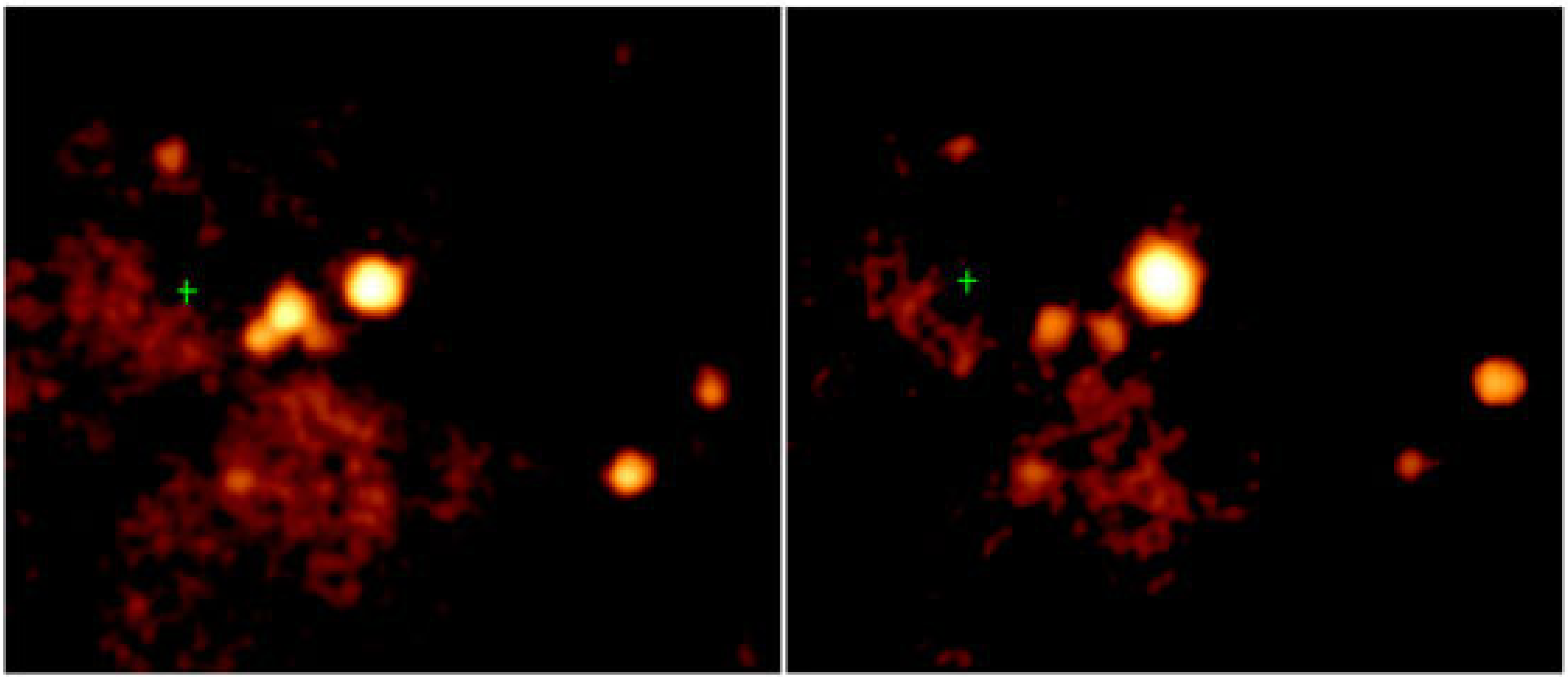


Some recent topics

- **Collision of stars and blackhole formation**

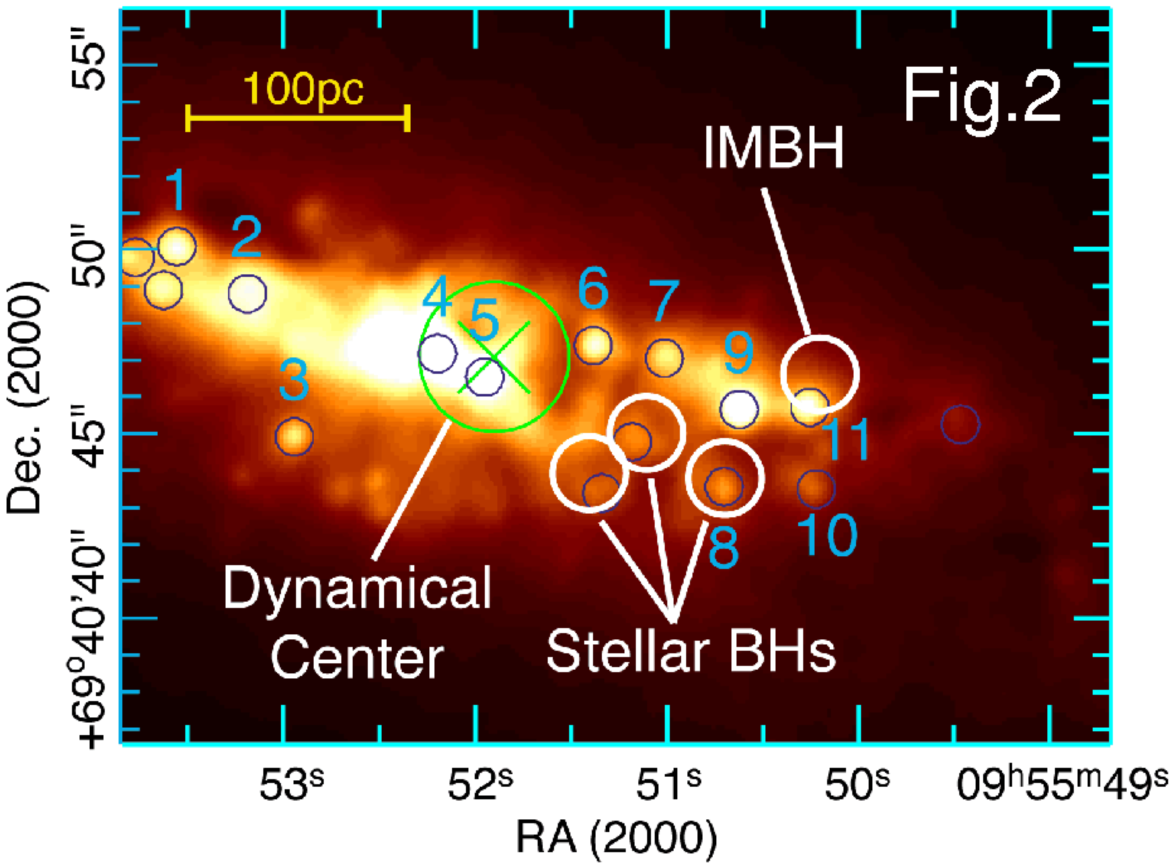
IMBH candidate in M82

Matsumoto *et al.* ApJL 547, L25



X-ray sources. Brightest one: Corresponds to 1000-solar-mass black hole (Intermidiate-mass BH)

Subaru observation



Host star cluster

McCrady et al. 2003 (astro-ph/0306373)
Cluster #11 (MGG-11)

- $\sigma_r = 11.4 \pm 0.8\mathrm{km/s}$
- half-light radius $1.2 \pm 0.17\mathrm{pc}$
- kinetic mass $3.5 \pm 0.7 \times 10^5 M_\odot$
- Age $\sim$ 10Myrs.

Very short relaxation time (less than 10Myrs)

A possible scenario

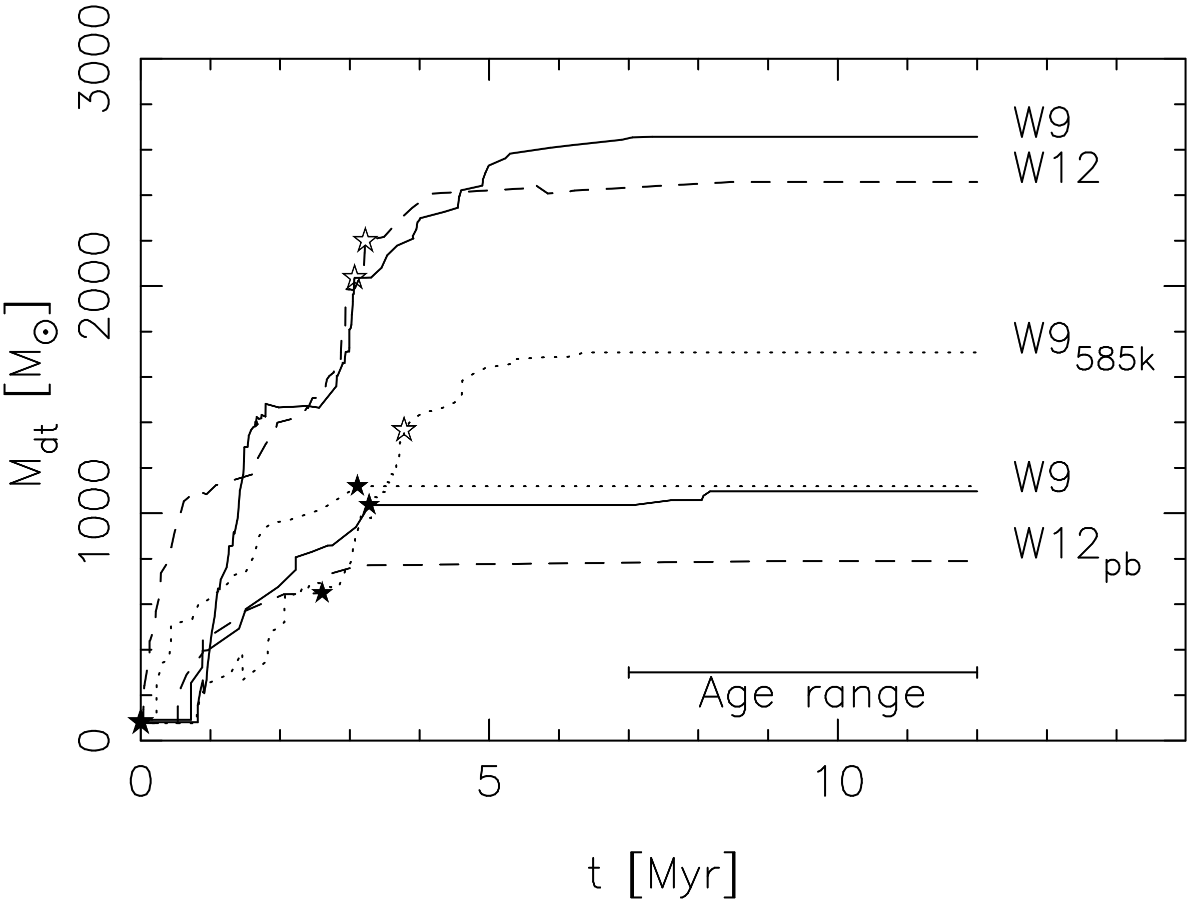
1. **Massive stars sink to the center through relaxation**

2. **Supermassive star forms through collision and merging**

3. **This star collapses to a BH**

Simulation

Portegies Zwart et al., Nature, Apr 15, 2004

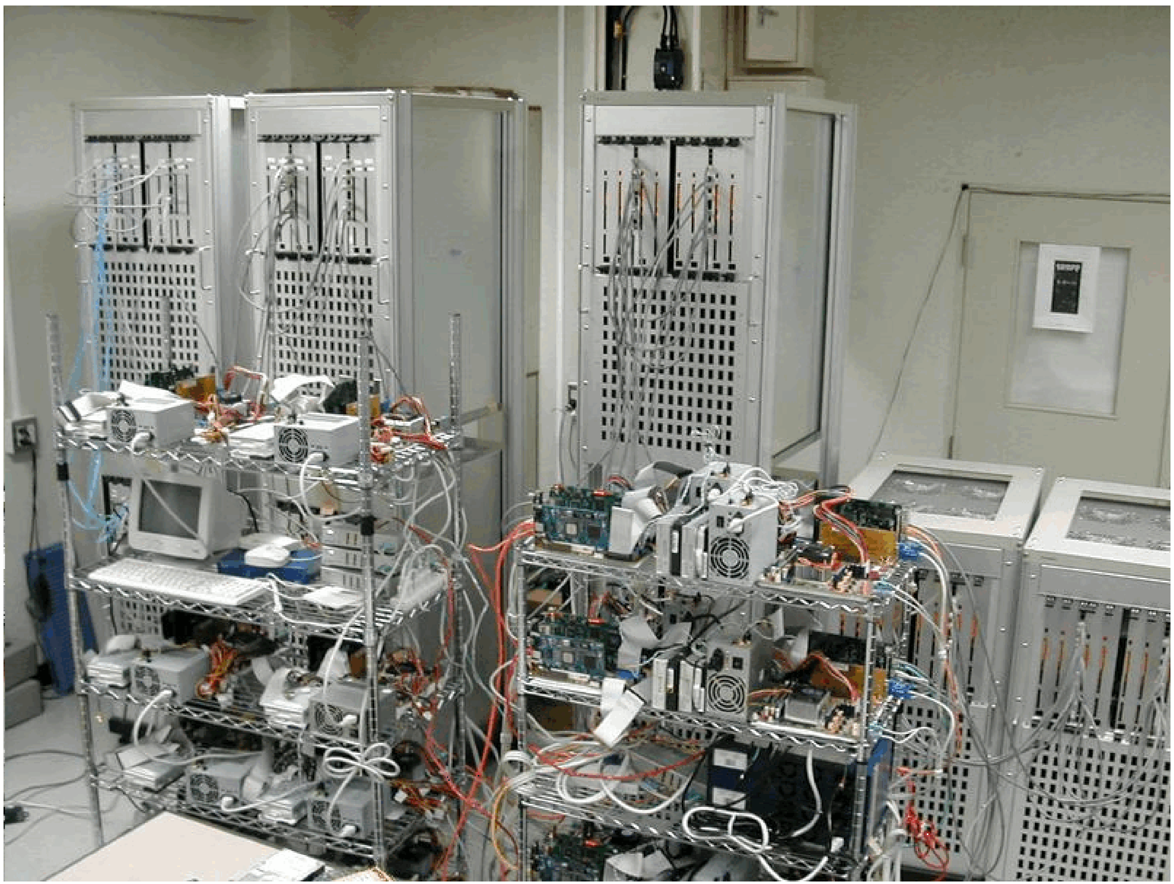


This scinario seems to work

GRAPE-6

Special-purpose computer for N-body simulations.

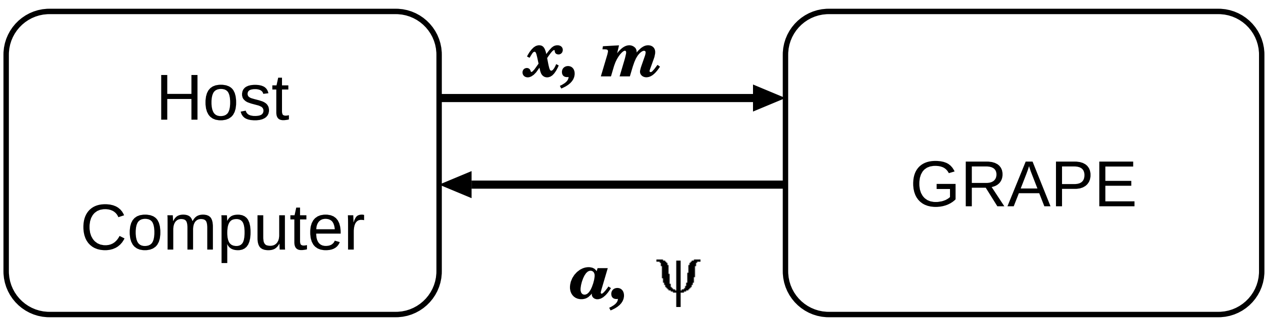
64 Tflops peak — World's fastest computer as of 2002.



Basic idea of GRAPE

Special-purpose hardware for force calculation
General-purpose host for all other calculation

$$\frac{d^2 \boldsymbol{x}_i}{dt^2} = \sum_{j \neq i} -Gm_j \frac{\boldsymbol{x}_i - \boldsymbol{x}_j}{|\boldsymbol{x}_i - \boldsymbol{x}_j|^3}$$

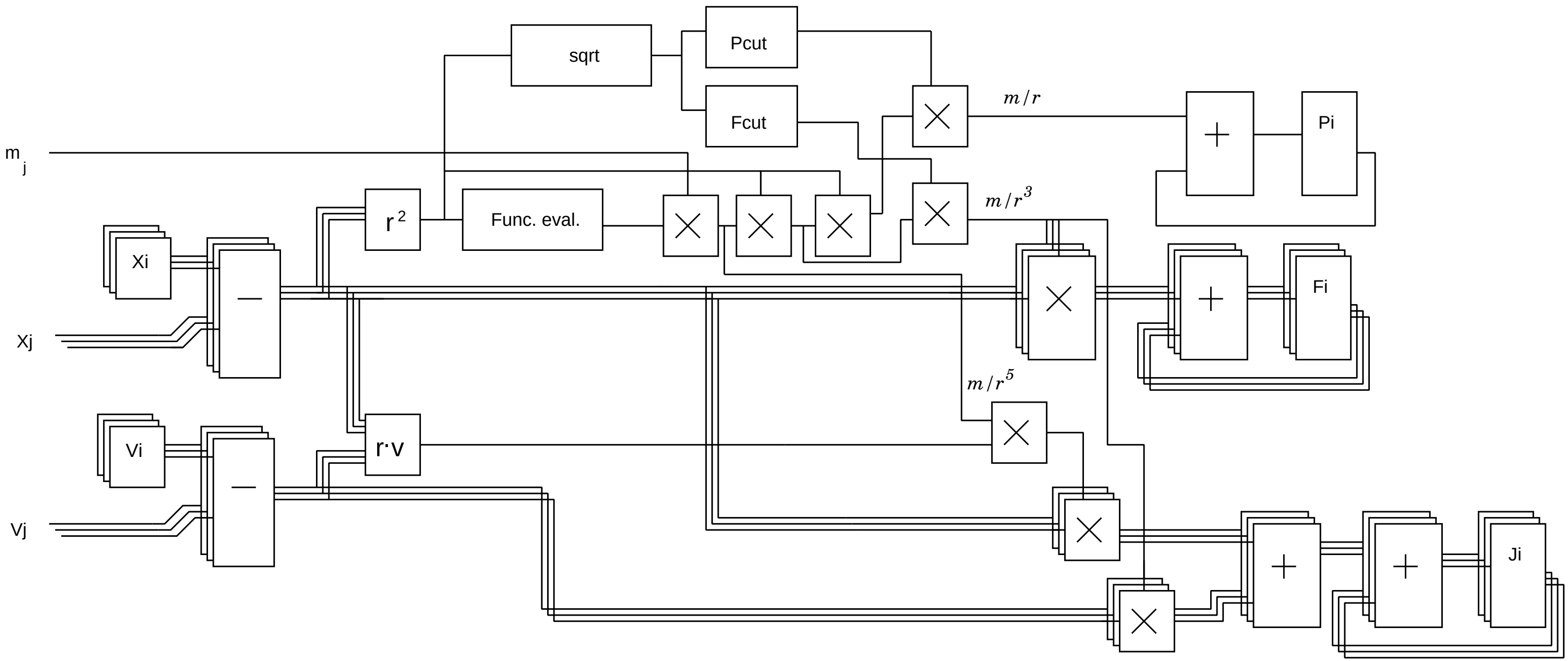


Time integration,
IO, etc
Flexibility

Force calculation

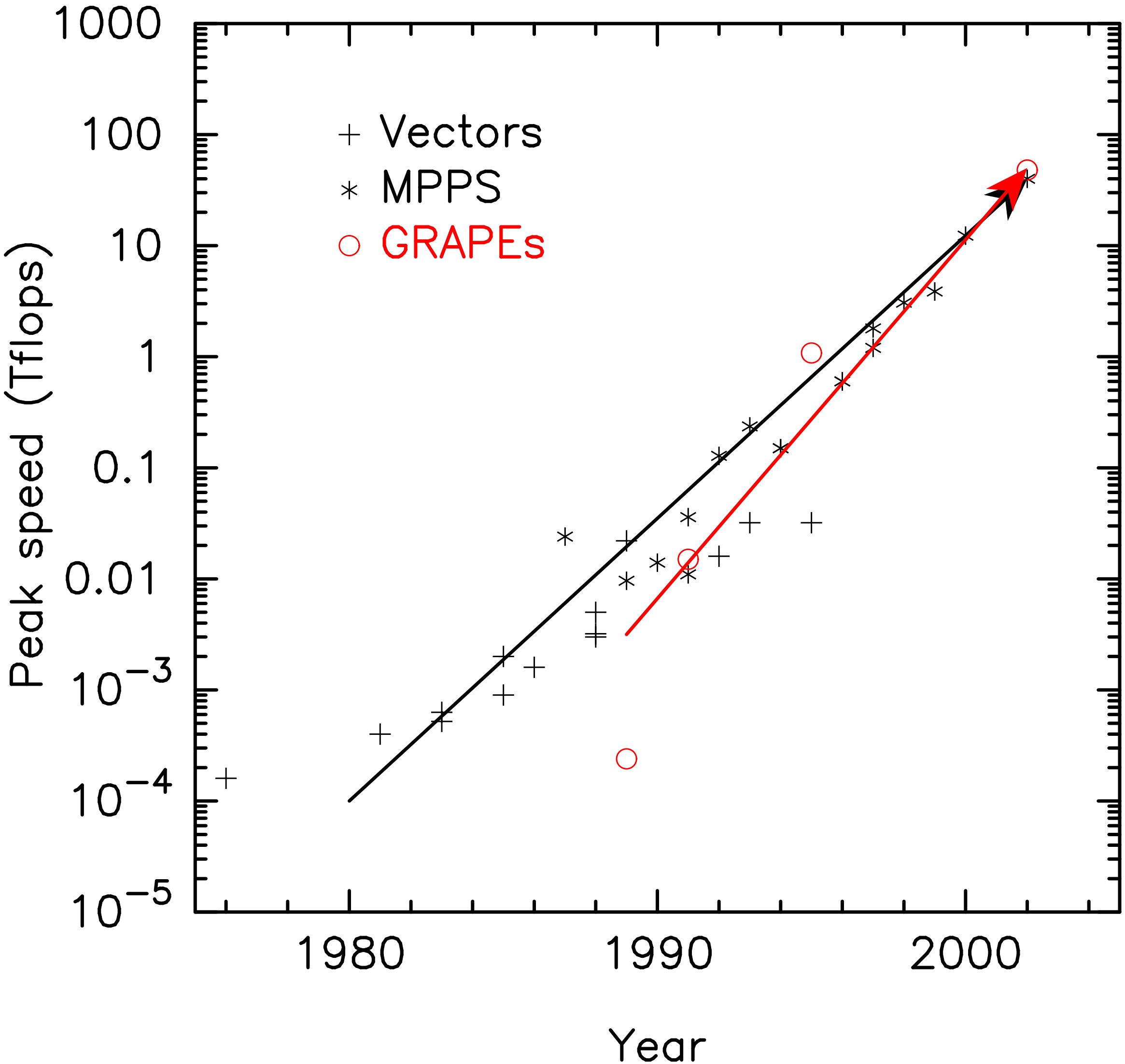
High performance

GRAPE-6 Processor pipeline



Calculates gravitational force, its first time derivative and potential.

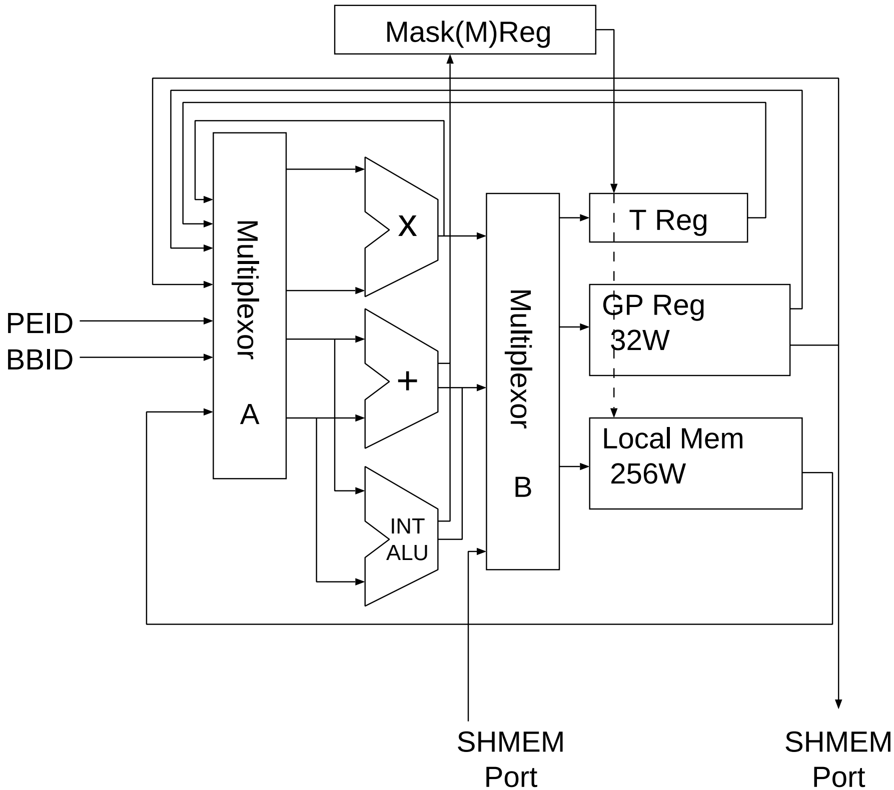
Evolution of GRAPE systems



Next-Generation GRAPE — GRAPE-DR

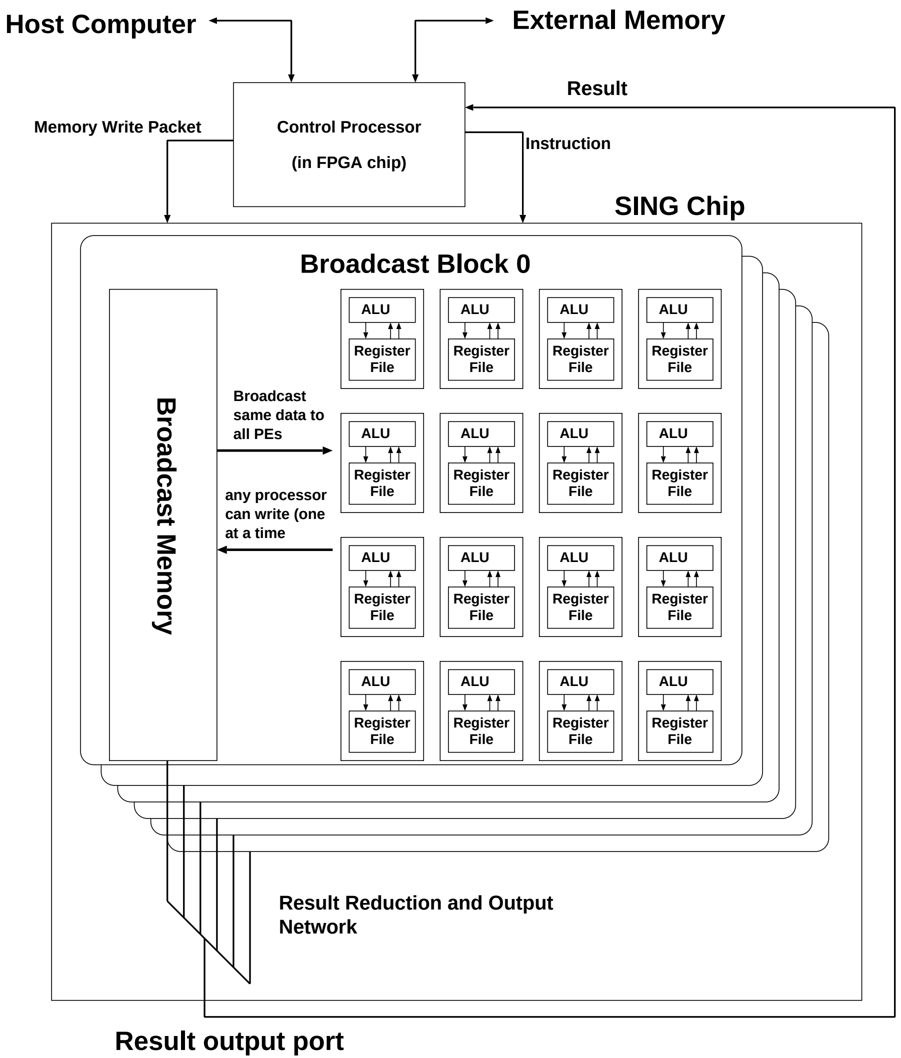
- Planned peak speed: 2 Pflops
- New architecture — wider application range than previous GRAPEs
- primarily to get funded
- No force pipeline. SIMD programmable processor
- Planned completion year: FY 2008 (early 2009)

Processor architecture



- **Float Mult**
- **Float add/sub**
- **Integer ALU**
- **32-word registers**
- **256-word memory**
- **communication port**

Chip structure



Collection of small processors.

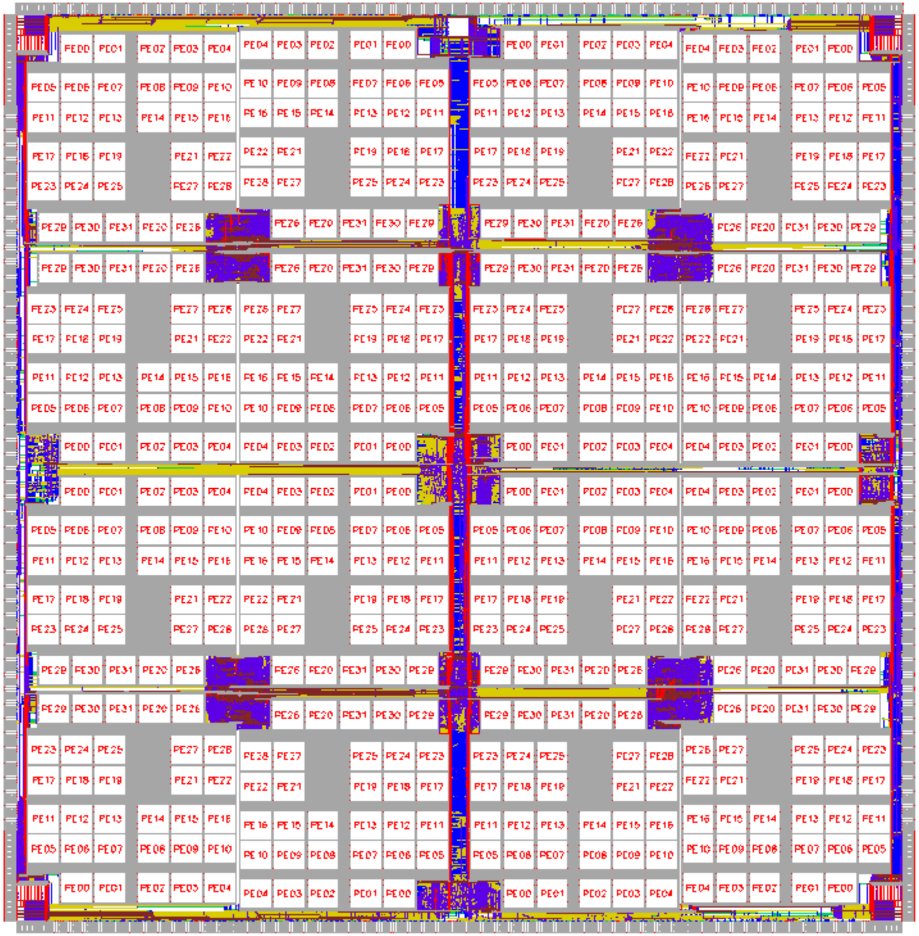
512 processors on one chip
500MHz clock

Peak speed of one chip: 0.5 Tflops (20 times faster than GRAPE-6).

Development status

Sample chip delivered May 2006

Chip layout



- **32PEs in 16 groups**
- **18mm by 18mm**

Prototype board

2nd prototype board. (Designed by Toshi Fukushige)

Difference from the 1st one:

PCI-Express x8 interface

On-board DRAM

Designed to run real applications

Summary

- Gravitational Many-body problem is fun
- Recently, numerical integration of N-body systems have become useful tool for modelling observations.
- Special-purpose computers help.