(Simplified) Mathematics of Coronavirus (SARS-CoV-2)

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Talk overview

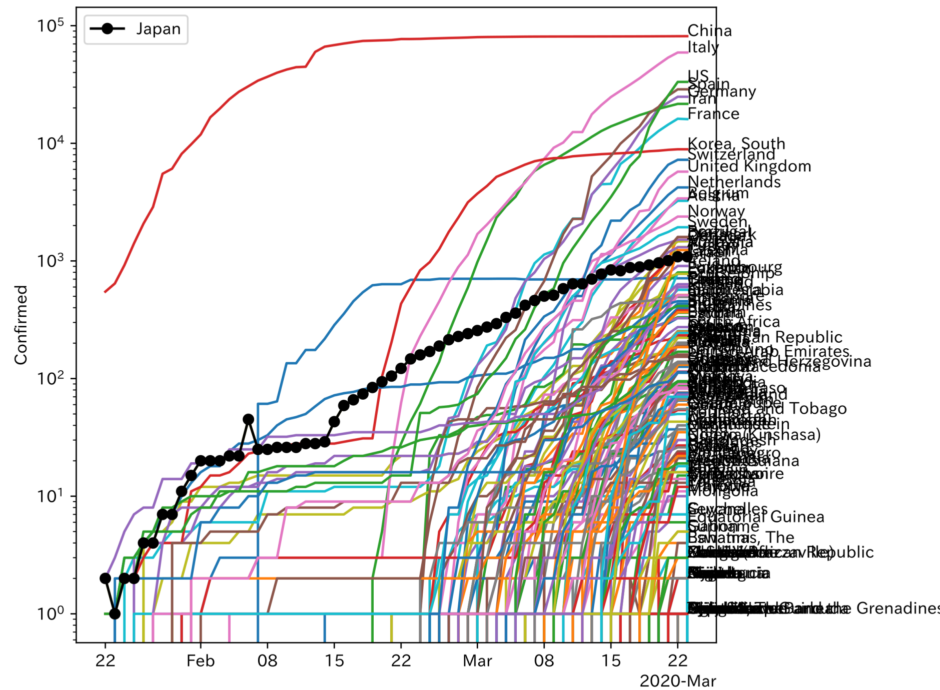
- What is happening?
- How we can understand the current situation?
- What should be done?

What is happening?

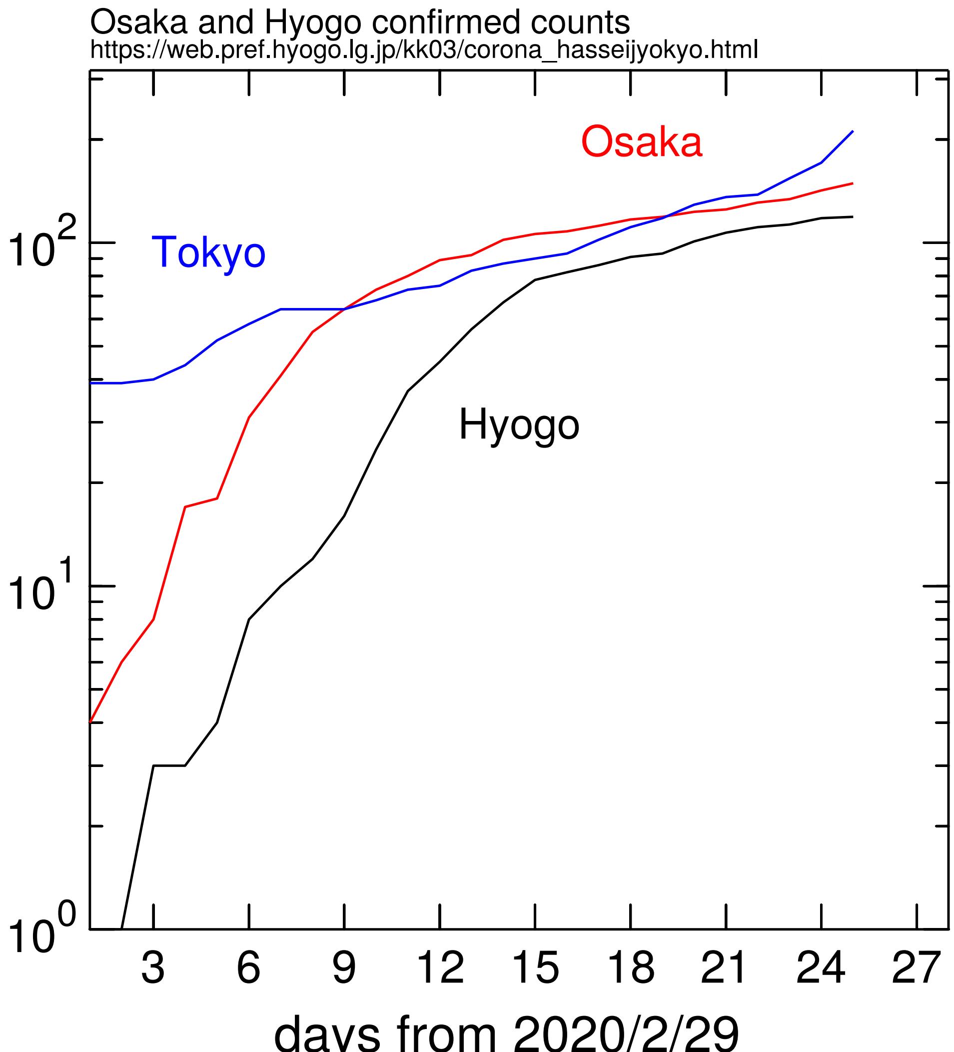
- In many countries, SARS-CoV-2 cases have been, and still are, increasing with the doubling time of 1.5-2 days.

- In China, South Korea and Qatar, the numbers have apparently stabilized.

- In Japan, the doubling timescale is around one week, if we believe governmental statistics (which I do not...)



What is happening in Japan?



Tokyo looks bad...

How we can understand the current situation?

The simplest mathematical model which takes into account the effect of finite population — the SIR model (Kermack and McKendrick 1927)

$\boldsymbol{S}$: Susceptible (those not infected yet)
$\boldsymbol{I}$: Infected
$\boldsymbol{R}$: Recovered (including deaths)

$$\begin{aligned} \boldsymbol{S} + \boldsymbol{I} + \boldsymbol{R} &= \boldsymbol{N}(\mathbf{const.}), && (1) \\ \frac{\boldsymbol{dI}}{\boldsymbol{dt}} &= \boldsymbol{\beta S I - \gamma I}, && (2) \\ \frac{\boldsymbol{dR}}{\boldsymbol{dt}} &= \boldsymbol{\gamma I}. && (3) \end{aligned}$$

Model assumptions

For a given population (*e. g.*, that of Japan),

- the increase rate of the infected people is proportional to the product of the number of infected and that of not yet infected people.
- Infected people recovers at the timescale of $1/\gamma$.

Very simple one-zone model.

Nondimensional equations

$$x = I/N, \quad y = R/N, \quad s = S/N = 1 - x - y. \tag{4}$$

$$\frac{dx}{dt} = ax(1 - x - y) - bx, \tag{5}$$

$$\frac{dy}{dt} = bx. \tag{6}$$

Time transformation(to make $b = 1$)

$$\frac{dx}{dt} = R_0 x(1 - x - y) - x, \tag{7}$$

$$\frac{dy}{dt} = x. \tag{8}$$

R_0 is called “The Basic Reproduction Number”.

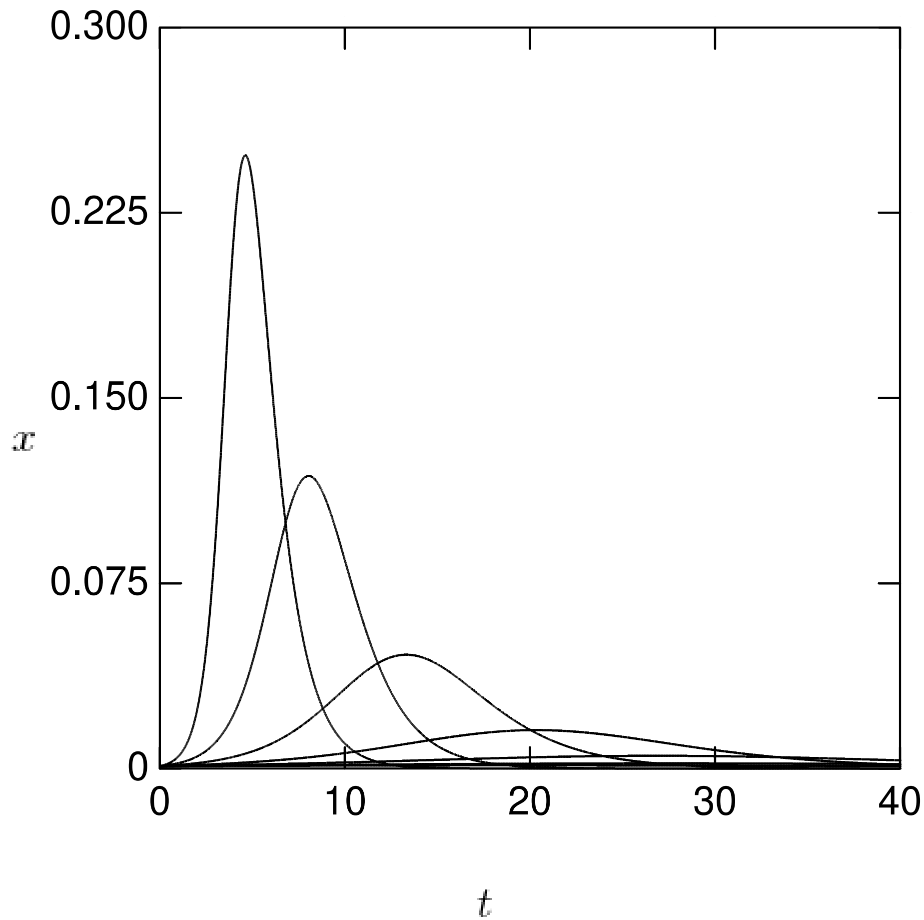
Limiting behavior

- If $R_0 < 1$, x decreases exponentially. In other words, the outbreak does not occur.
- If $R_0 > 1$ and $x \ll 1$, x increases exponentially. (Outbreak)
- If $R_0 > 1$, x will increase until $R_0(1 - x - y)$ becomes less than unity. After than x approaches to zero, and y converges to some constant which depends on R_0.

Examples of solutions

Plot of x: The number of infected people
R_0: 1.025, 1.05, 1.1, 1.2, 1.4, 1.8, 2.6.
$x_0 : 10^{-3}$

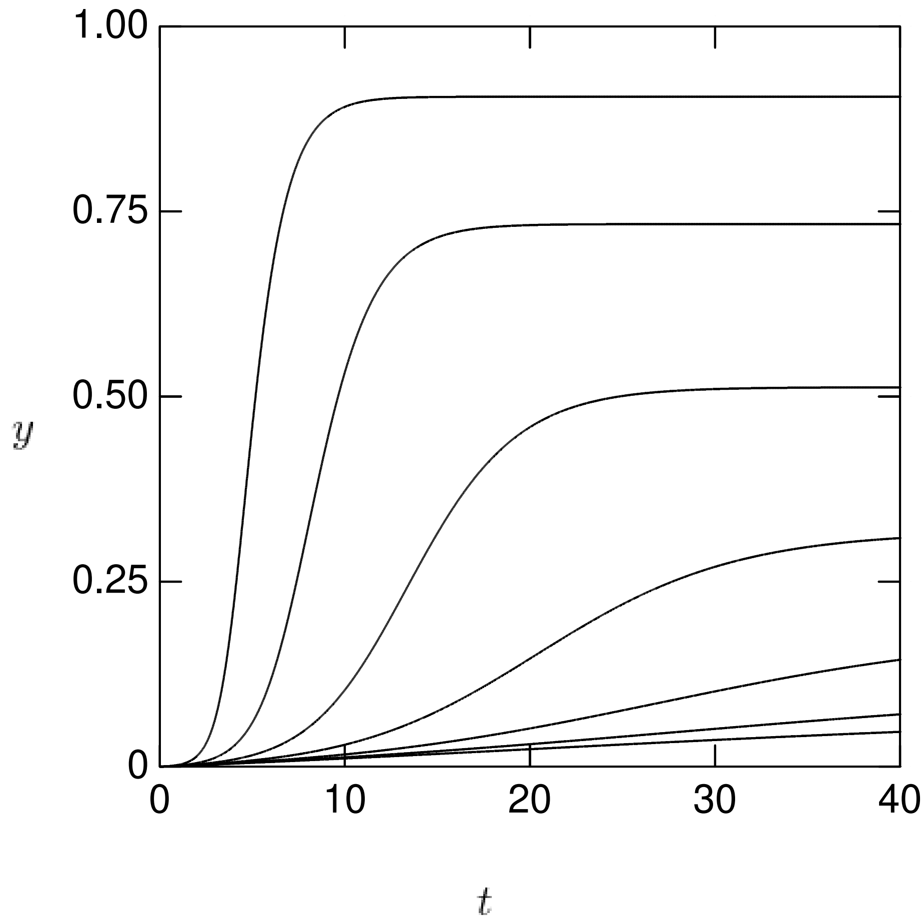
If $R_0 \sim 1$, x remains small.
So looks okay?



Number of "recovered"

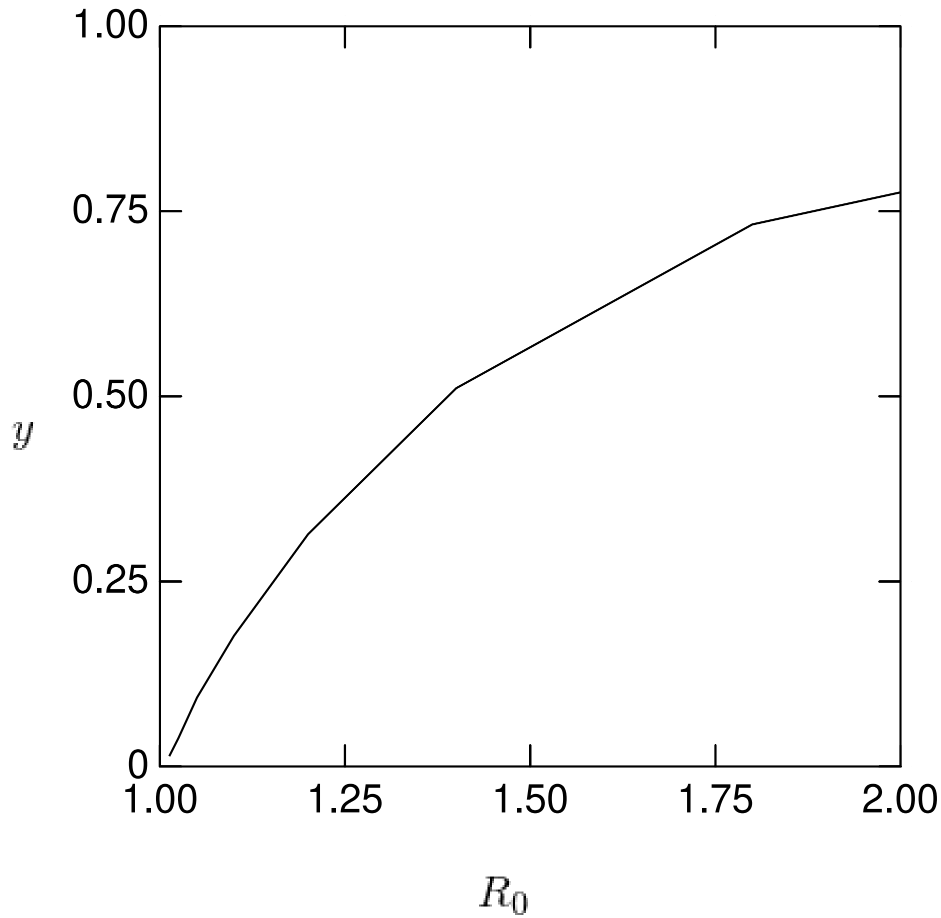
$\boldsymbol{R_0}$: 1.025, 1.05, 1.1, 1.2, 1.4, 1.8, 2.6.

Even if $\boldsymbol{R_0 \sim 1}$, $\boldsymbol{y}$ is not very small.



Number of “recovered” and R_0

We should be able to get y in the limit of $R_0 \to 1$ (not yet done)
Note: y multiplied by deaths rate = total number of deaths.
Death rate$\sim$ 2% (age averaged) or higher...



It is clear that we should make R_0 less than unity.

What should be done?

Question: How can we achieve $R_0 < 1$?
China: lockdown
South Korea: combination of aggressive testing and moderate limitation of social activities(?)

If we can find a fair fraction of infected people (not necessarily all), we can reduce R_0. But can we?

What can we do?

- Even when $R_0 > 1$, we can try to reduce our individual risk to be infected. After the outbreak finished, average risk to be infected is the asymptotic value of y. We can make our individual risk to be smaller. Avoid meeting people or going to crowded area. Using masks and handwashing would also help.

- If everybody (or a majority of people) reduced their individual risks, R_0 is reduced.

- Thus, here our individual effort to reduce personal risk would help our society to survive.