(Simplified) Mathematics of Coronavirus (SARS-CoV-2)

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Talk overview

- What is happening?
- How we can understand the current situation?
- What should be done?
What is happening?

- In many countries, SARS-CoV-2 cases have been, and still are, increasing with the doubling time of 1.5-2 days.

- In China, South Korea and Qatar, the numbers have apparently stabilized.

- In Japan, the doubling timescale is around one week, if we believe governmental statistics (which I do not...)
What is happening in Japan?

Osaka and Hyogo confirmed counts
https://web.pref.hyogo.lg.jp/kk03/corona_hasseijyokyo.html

Tokyo looks bad...
How we can understand the current situation?

The simplest mathematical model which takes into account the effect of finite population — the SIR model (Kermack and McKendrick 1927)

\( S \): Susceptible (those not infected yet)
\( I \): Infected
\( R \): Recovered (including deaths)

\[ S + I + R = N \text{(const.)}, \]
\[ \frac{dI}{dt} = \beta SI - \gamma I, \]
\[ \frac{dR}{dt} = \gamma I. \]
Model assumptions

For a given population (e.g., that of Japan),

- the increase rate of the infected people is proportional to the product of the number of infected and that of not yet infected people.

- Infected people recovers at the timescale of $1/\gamma$.

Very simple one-zone model.
Nondimensional equations

\[ x = I/N, \quad y = R/N, \quad s = S/N = 1 - x - y. \]

\[ \frac{dx}{dt} = ax(1 - x - y) - bx, \quad (5) \]

\[ \frac{dy}{dt} = bx. \quad (6) \]

Time transformation (to make \( b = 1 \))

\[ \frac{dx}{dt} = R_0x(1 - x - y) - x, \quad (7) \]

\[ \frac{dy}{dt} = x. \quad (8) \]

\( R_0 \) is called "The Basic Reproduction Number".
Limiting behavior

- If $R_0 < 1$, $x$ decreases exponentially. In other words, the outbreak does not occur.

- If $R_0 > 1$ and $x \ll 1$, $x$ increases exponentially. (Outbreak)

- If $R_0 > 1$, $x$ will increase until $R_0(1 - x - y)$ becomes less than unity. After that $x$ approaches to zero, and $y$ converges to some constant which depends on $R_0$. 
Examples of solutions

Plot of $x$: The number of infected people

$R_0$: 1.025, 1.05, 1.1, 1.2, 1.4, 1.8, 2.6.

$x_0 : 10^{-3}$

If $R_0 \sim 1$, $x$ remains small.
So looks okay?
Number of “recovered”

$R_0$: 1.025, 1.05, 1.1, 1.2, 1.4, 1.8, 2.6.

Even if $R_0 \sim 1$, $y$ is not very small.
We should be able to get $y$ in the limit of $R_0 \to 1$ (not yet done)
Note: $y$ multiplied by deaths rate = total number of deaths.
Death rate $\sim 2\%$ (age averaged) or higher...

It is clear that we should make $R_0$ less than unity.
What should be done?

Question: How can we achieve $R_0 < 1$?
China: lockdown
South Korea: combination of aggressive testing and moderate limitation of social activities(?)

If we can find a fair fraction of infected people (not necessarily all), we can reduce $R_0$. But can we?
What can we do?

- Even when $R_0 > 1$, we can try to reduce our individual risk to be infected. After the outbreak finished, average risk to be infected is the asymptotic value of $y$. We can make our individual risk to be smaller. Avoid meeting people or going to crowded area. Using masks and handwashing would also help.

- If everybody (or a majority of people) reduced their individual risks, $R_0$ is reduced.

- Thus, here our individual effort to reduce personal risk would help our society to survive.