Numerical Astrophysics by GRAPE

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- Basic Idea of GRAPE
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  - Formation and structure of ellipticals with central massive BHs
  - Demagnification of Type Ia SNe by multiple gravitational lensing

N-body Simulation

$$\frac{d\mathrm{x}_i}{dt} = \sum_{j \neq i} -Gm_j \frac{\mathrm{x}_i - \mathrm{x}_j}{|\mathrm{x}_i - \mathrm{x}_j|^3}$$

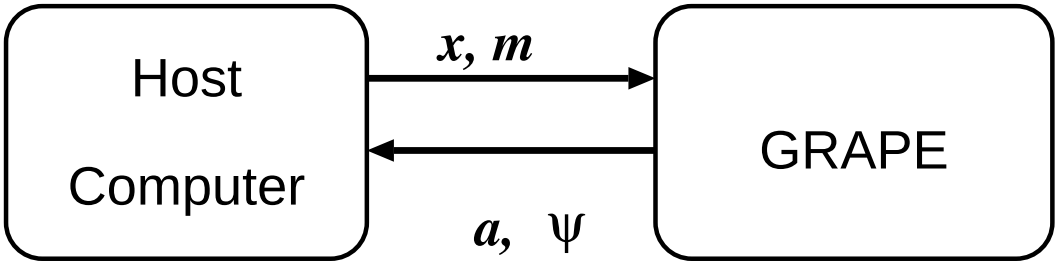
describes the dynamics of

- Solar system
- Open clusters
- Globular clusters
- Galaxies
- Groups and clusters of galaxies
- Large scale structures

Main difficulty: Calculation cost $\propto N^2$

Basic structure of GRAPE

Special-purpose hardware for force calculation
General-purpose host for all other calculation



- Special-purpose hardware
  - Hardwired logic → No software needed
  - Pipelined processor → High performance
- General-purpose host
  - Programmable in high-level languages
  - existing programs can be used with small modifications
  - can be used with Barnes-Hut tree/Aarseth scheme

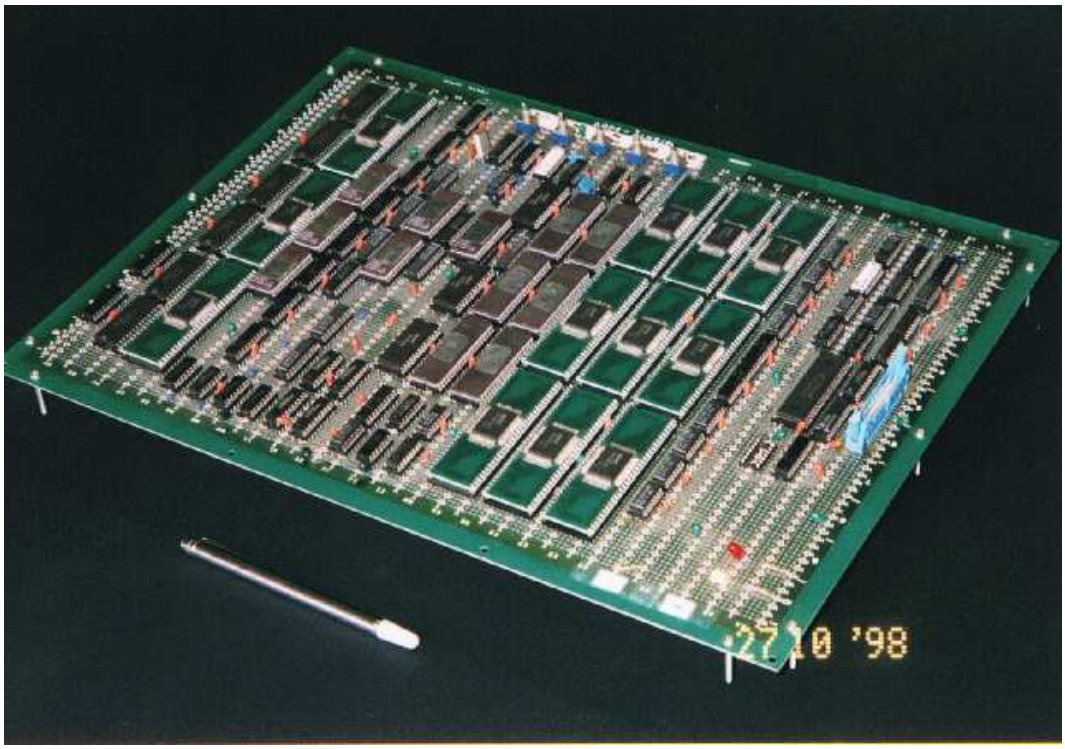
The GRAPE-1 Board

(1989)

Design	started:	May 1989
	finished:	July 1989
Hardware	completed:	September 1989

4 months $\times$ 1.5 persons

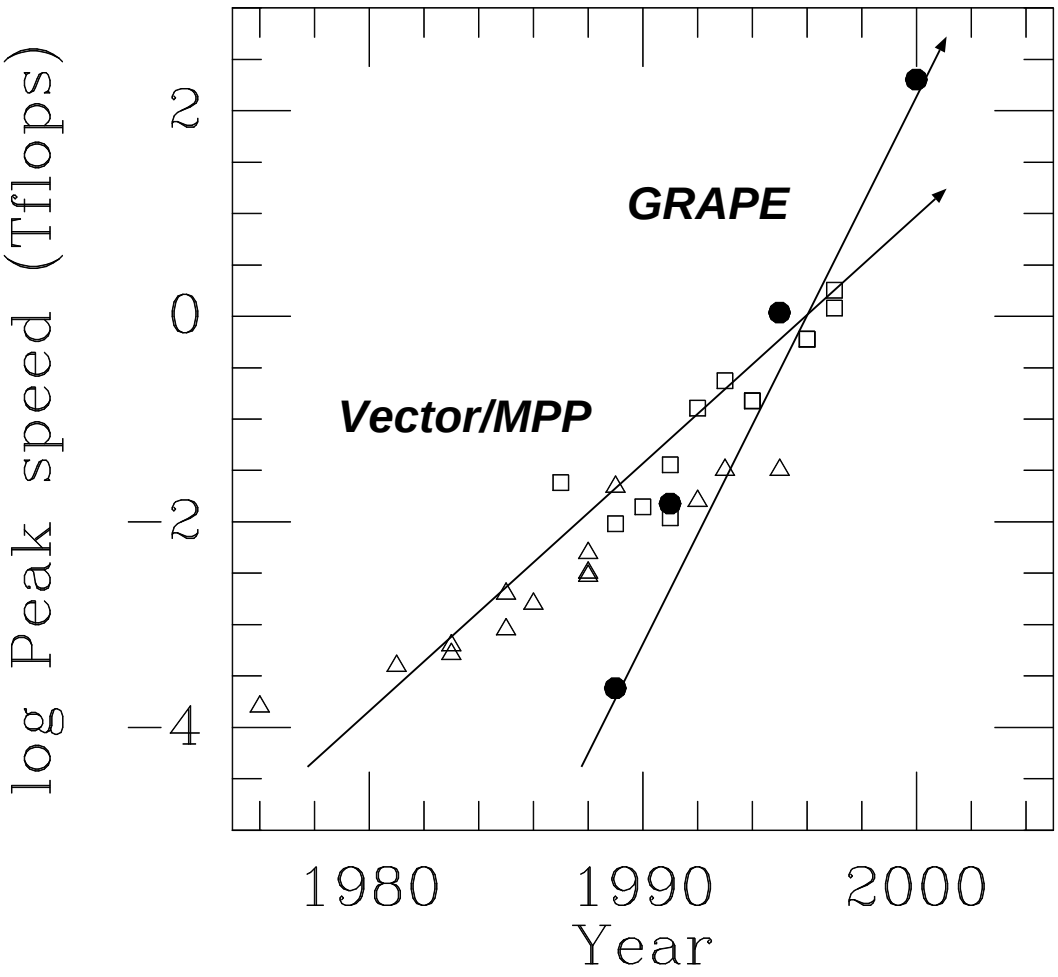
200K yens



History of the project

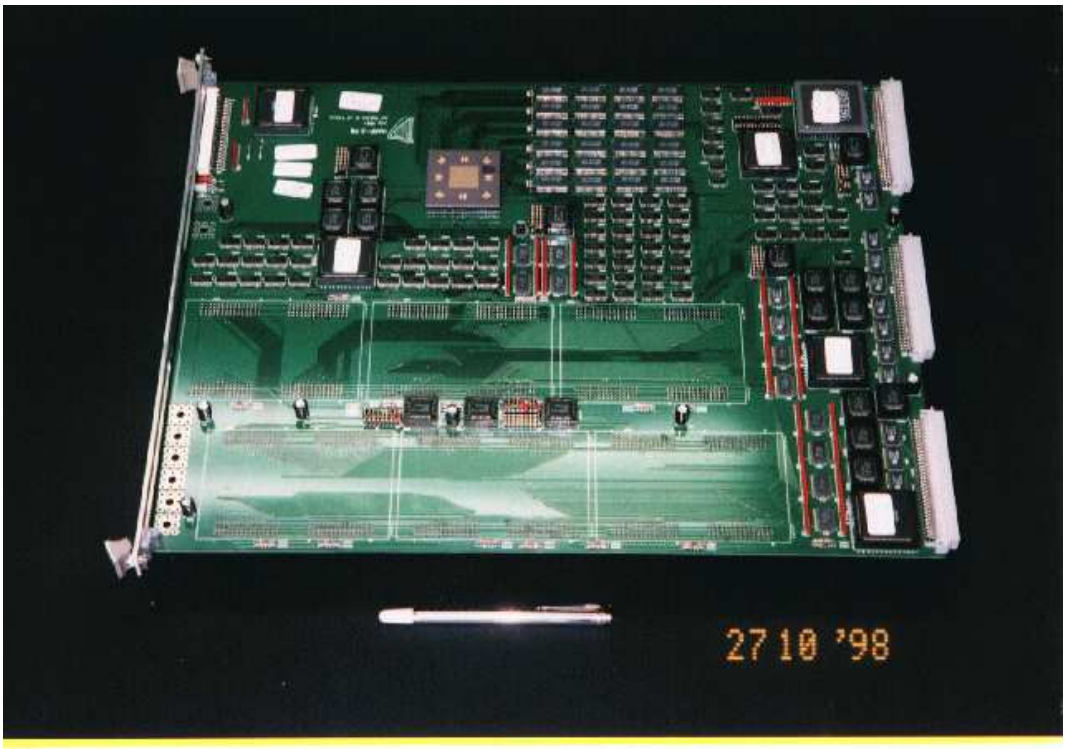
GRAPE-1	(89/4 — 89/10)	120 Mflops, low accuracy
GRAPE-2	(89/8 — 90/5)	40 Mflops, high accuracy(32bit/64bit)
GRAPE-1A	(90/4 — 90/10)	240 Mflops, low accuracy
GRAPE-3	(90/9 — 91/9)	14 Gflops, high accuracy
GRAPE-2A	(91/7 — 92/5)	180 Mflops, high accuracy(32bit/64bit)
HARP-1	(92/7 — 93/3)	180 Mflops, high accuracy(32bit/64bit)
		Hermite scheme
GRAPE-3A	(92/1 — 93/7)	6 Gflops/board
		some 80 copies are installed
GRAPE-4	(92/7 — 95/7)	1 Tflops, high accuracy (32bit/64bit)
		Some 10 copies of small machines
MD-GRAPE	(94/7 — 95/4)	1Gflops/chip, high accuracy
		programmable interaction
GRAPE-5	(96/4 — 99/4)	5 Gflops/chip, 40 Gflops/board
GRAPE-6	(97/8 —)	40 Gflops/chip, $>$ 100 Tflops

Evolution of GRAPE systems

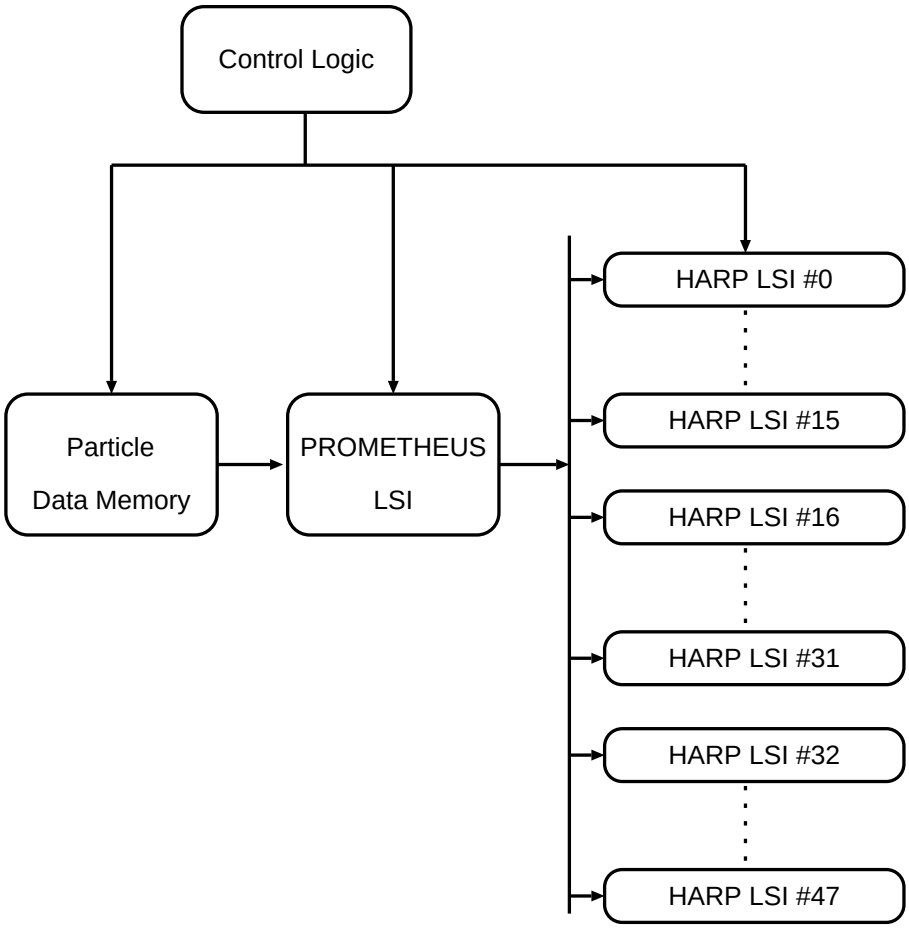


GRAPE-4

GRAPE-4 processing board



GRAPE-4 processing board

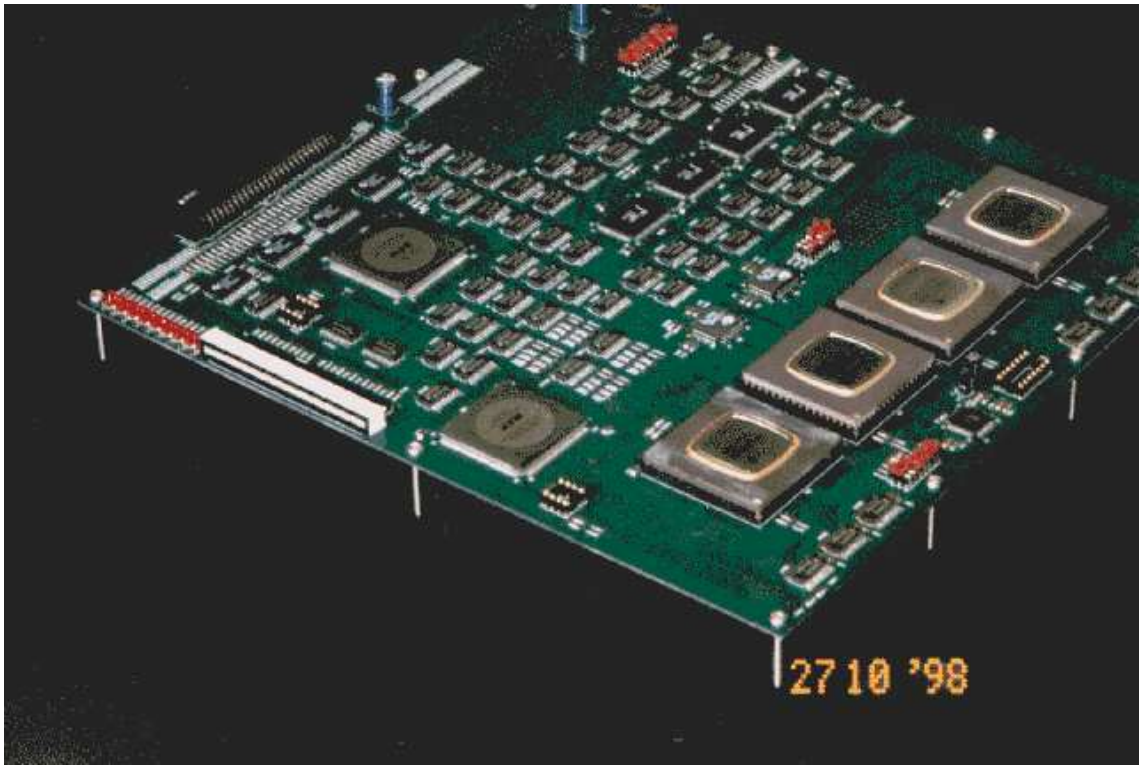


Many pipeline LSIs share one memory unit, calculating the forces on different particles.
→ Massively parallel system with small parts count, small physical size, and (relatively) low cost.

GRAPE-5

- Successor of GRAPE-3
- Improved calculation speed ($> 8\times$ per chip)
- Peak speed 40 Gflops /board
- Improved communication speed ($>$ 50MB/s)
- Support for periodic boundary condition

Prototype board (4 chips)



GRAPE-5 Availability

- First delivery in this summer (I hope)
- Price/board $\sim$ 15k$ (plus handling fee of trading company, plus ...)
- Host requirement: PCI interface.
- Recommended host: DEC (Compaq) Alpha with Digital UNIX.

GRAPE-6

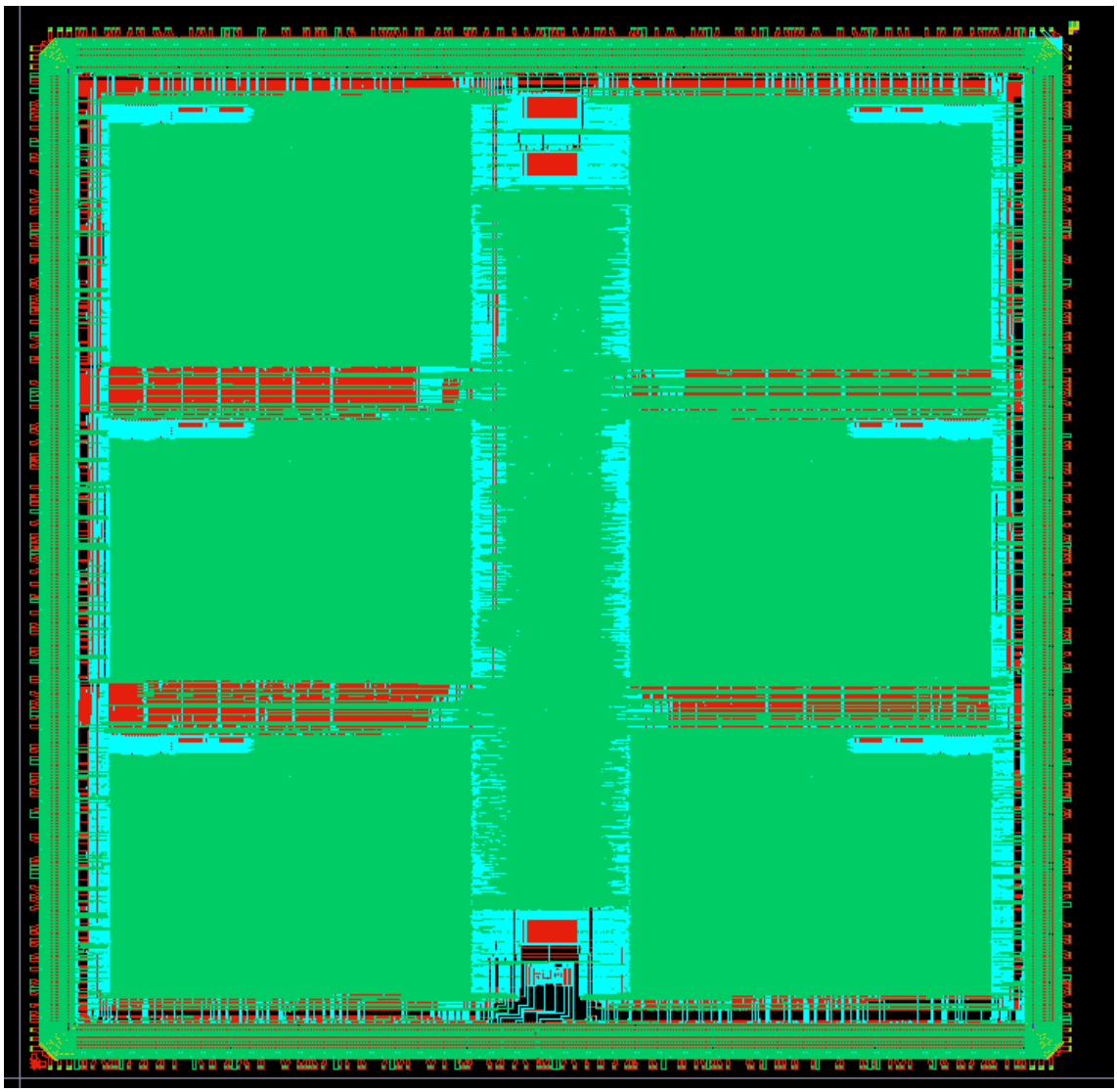
- Successor of GRAPE-4
- Improved calculation speed ($> 70\times$ per chip)
- Peak speed $>$ 100Tflops
- Improved communication speed ($>$ 2GB/s)
- Support for periodic boundary condition
- Chip sample arrived (not tested yet...)

- 16 clusters
- 16 processor boards per cluster
- 8 processor modules per board
- 2 chips per modules
- 6 pipelines per chip
- 60 arithmetic units per pipeline

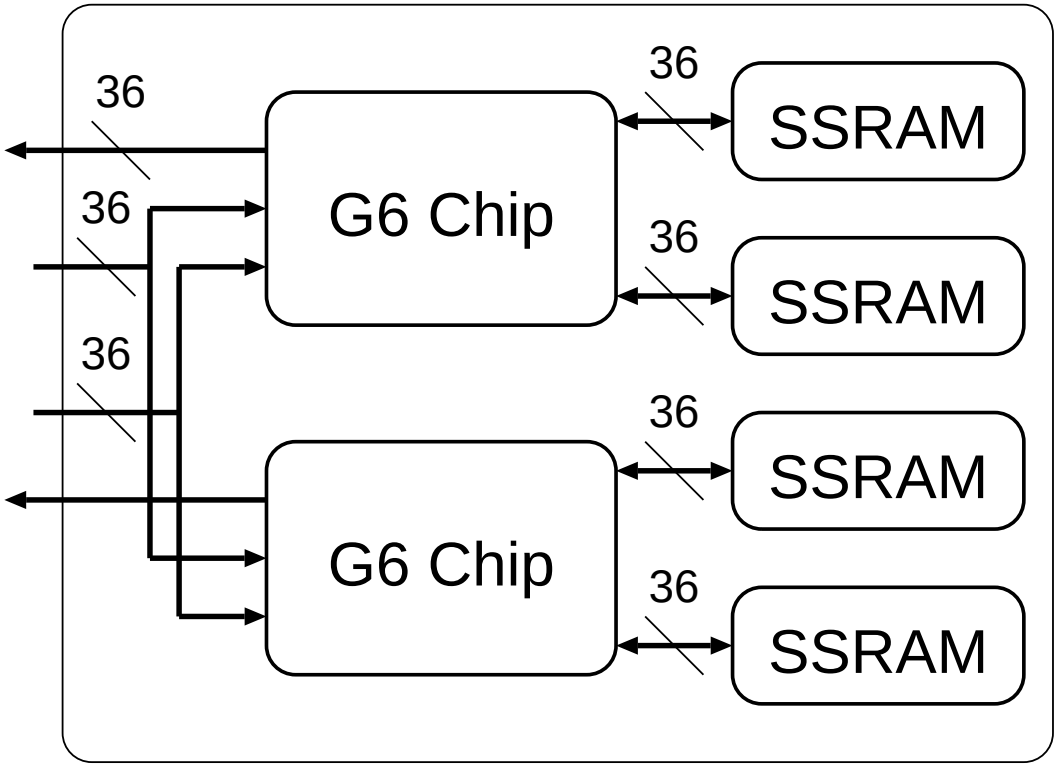
In total,1.5×10^6 Arithmetic units

GRAPE-6 Processor Chip

- 0.25 μm custom design (Toshiba TC240E, 1.8M gates)
- 2.5 V power supply, 100 MHz operation (worst case)
- 6 pipelines in a chip
- $>$ 30 Gflops (equivalent) per chip
- system clock: 25 MHz
- Single chip equivalent to single GRAPE-4 board

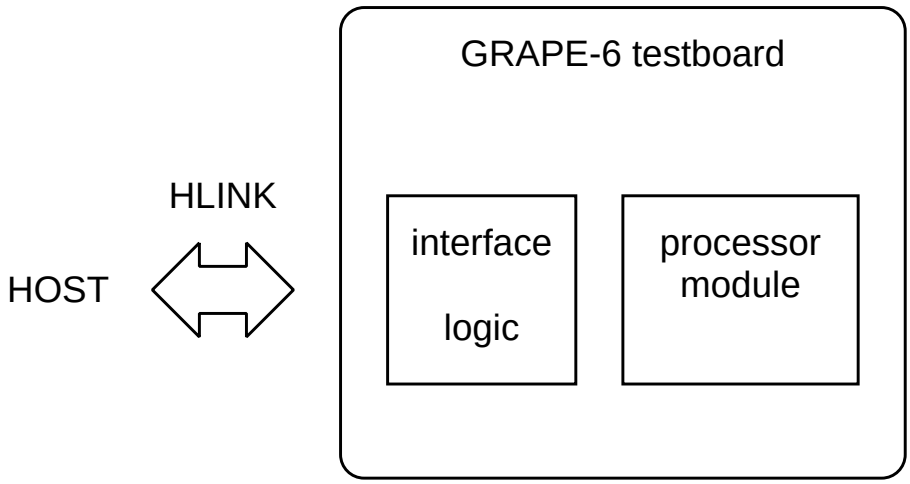


GRAPE-6 processor module



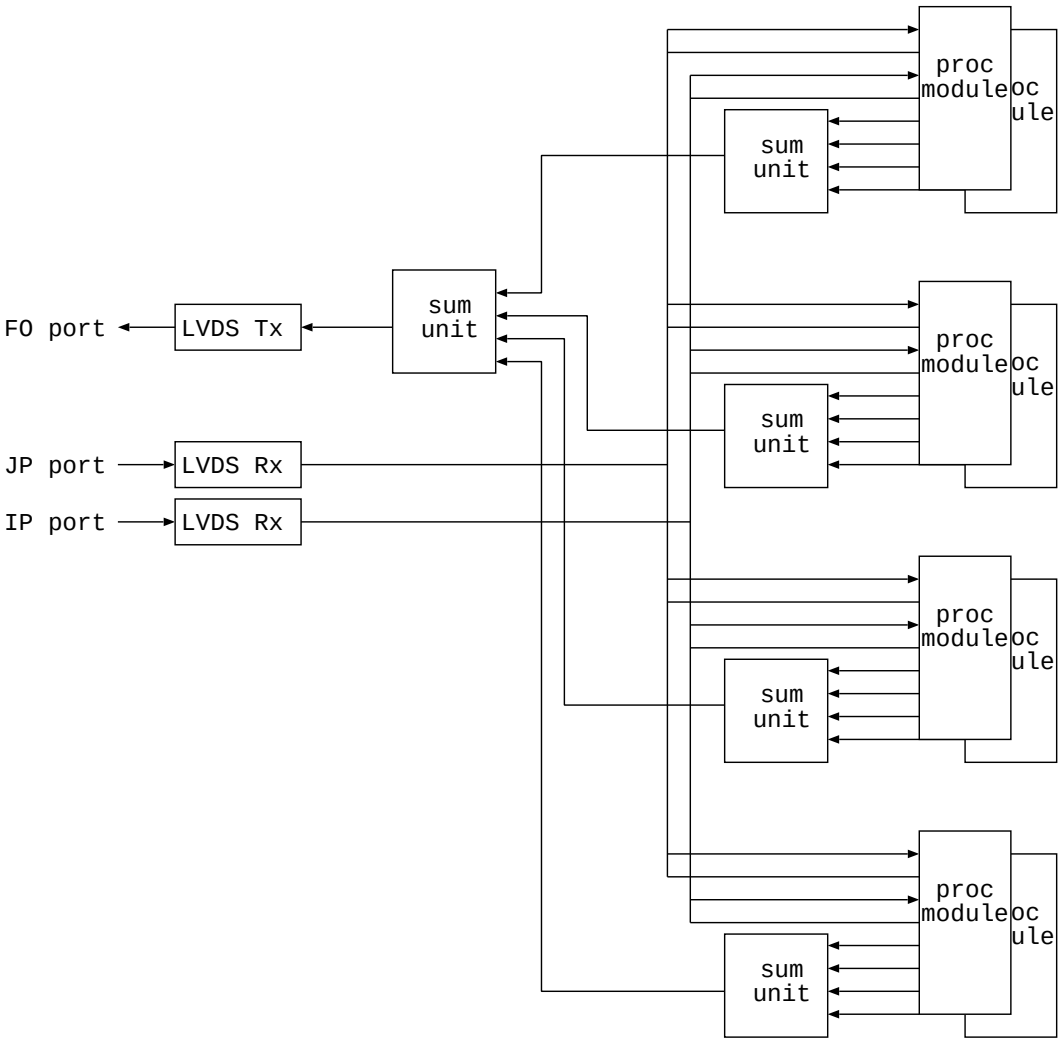
- Two processor chips + Memory chips.
- Integrates all functions of GRAPE-6.

The evaluation board



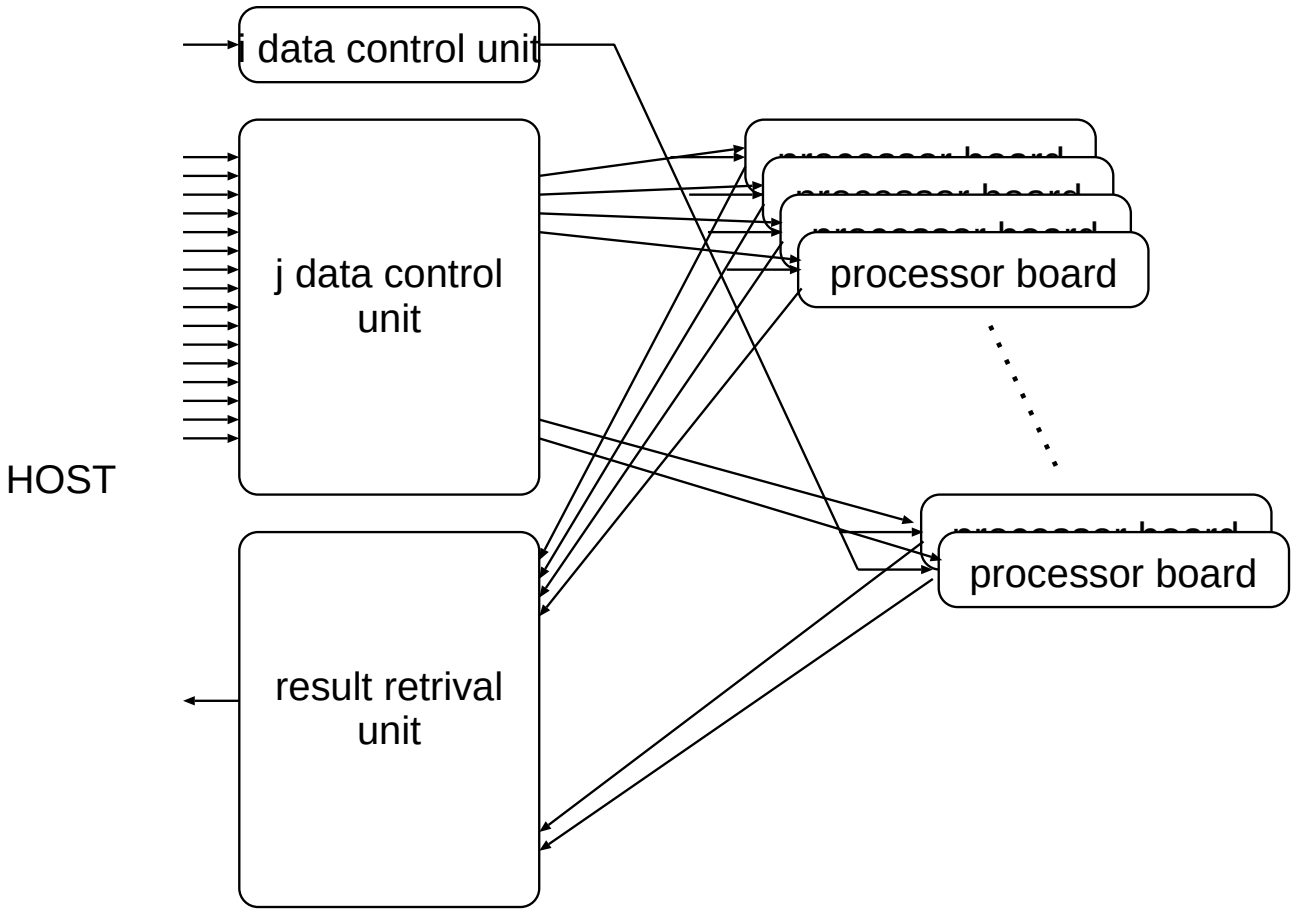
- Single module with PCI interface
- PCI interface same as GRAPE-4/5
- Arrived in March. Currently under test.

The GRAPE-6 Processor board



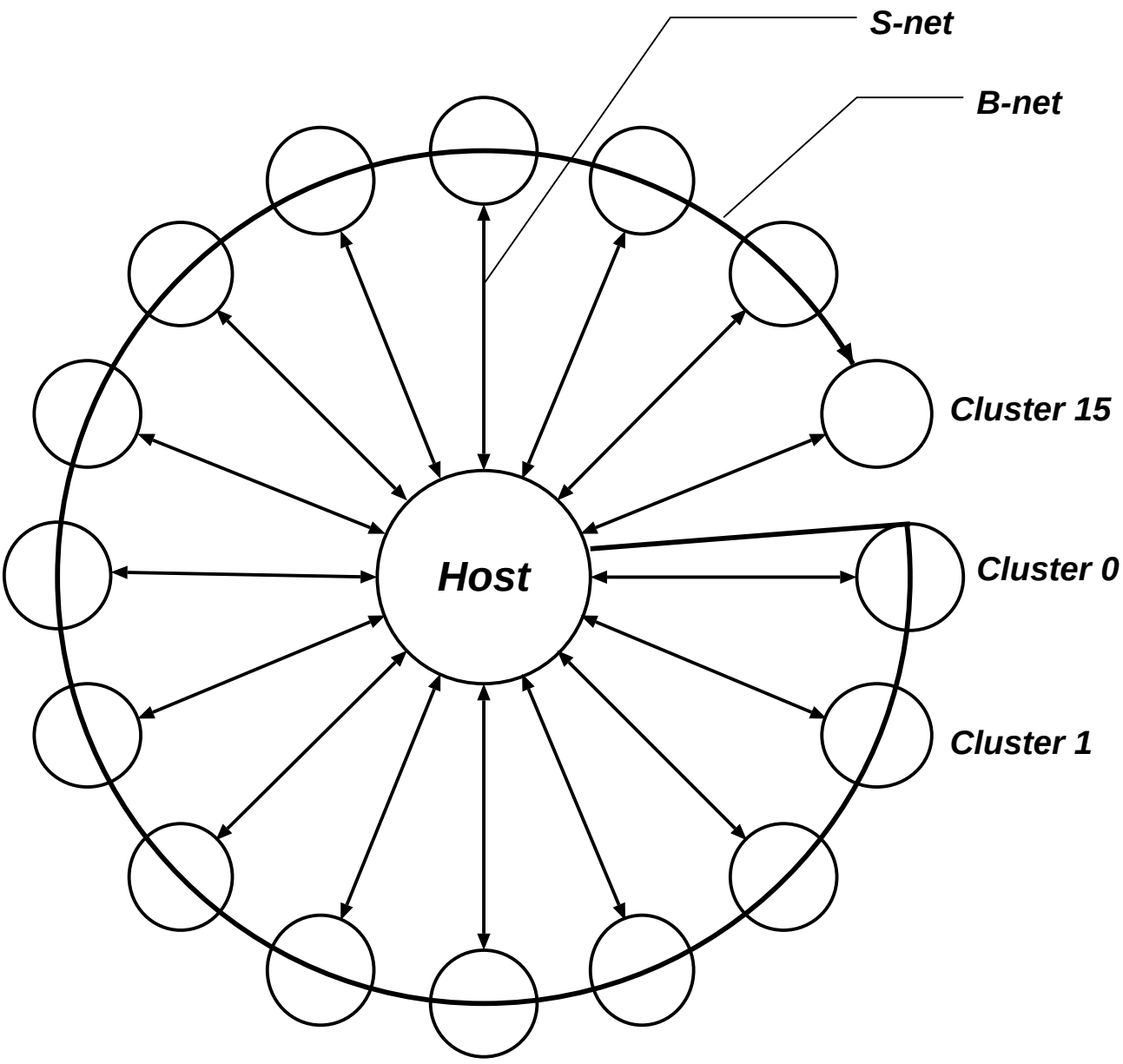
- Single board with 16-chips
- Semi-serial (LVDS) interface (operates at 350MHz)

Structure of a cluster



- 16 processor boards
- 3 dedicated communication networks

Structure of the total system



- 16 clusters
- 256 chips/cluster

Summary of GRAPE project

- A project to develop and use special-purpose computer for astrophysical N-body simulations.
- First custom LSI version, GRAPE-3, is still used worldwide.
- Completed the first Teraflops machine in 1995
- GRAPE-5, the successor of GRAPE-3, is completed.
- GRAPE-6, the successor of GRAPE-4, will be up and running soon.

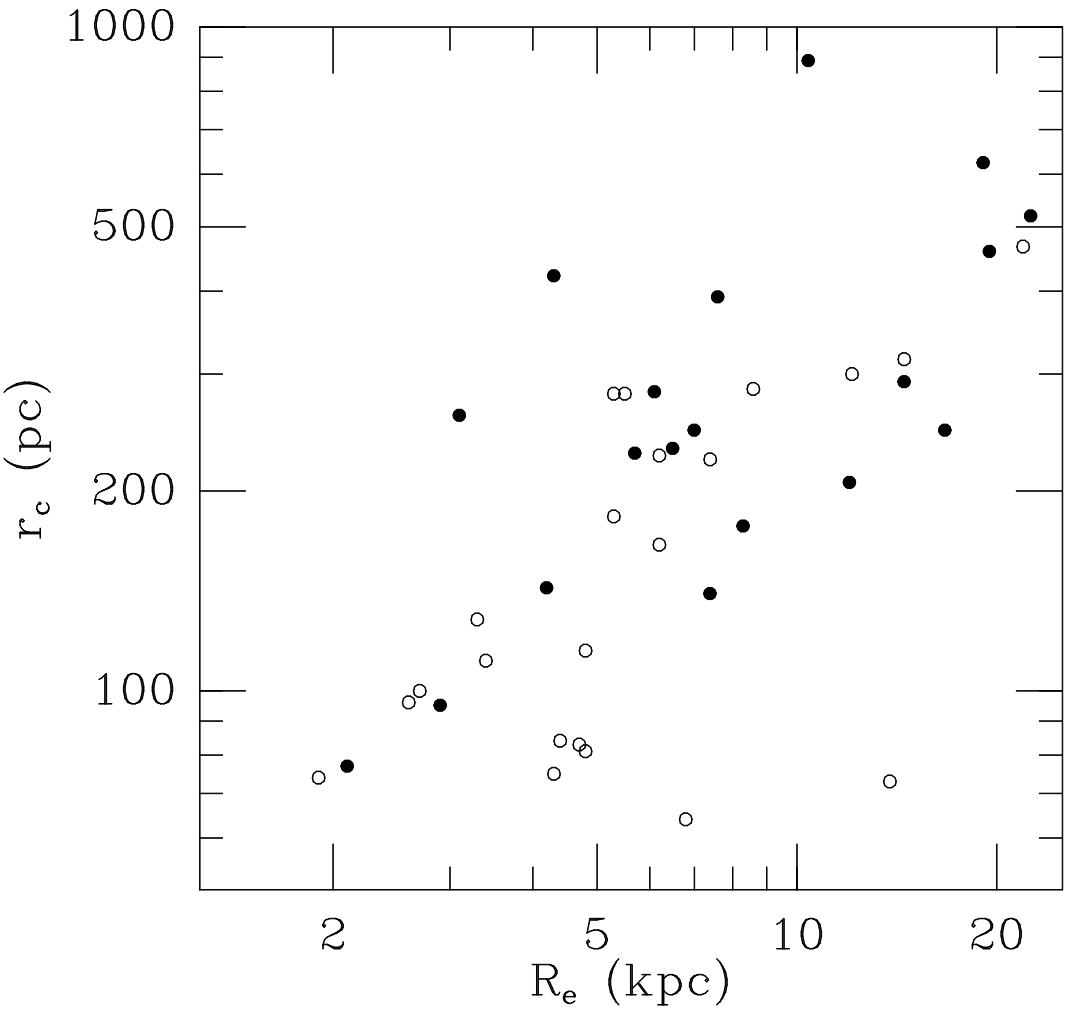
Some resent results with GRAPE

- Formation and structure of ellipticals with central massive BHs
- Demagnification of Type Ia SNe by multiple gravitational lensing

Ellipticals with central massive BHs

The central structure of ellipticals has been a mystery

The pre-HST view (Data from Lauer 1985)



- Ellipticals have cores
- $r_c \propto R_e$

Theories predict...

- No core (isothermal cusps) — dissipational collapse
- r_c = const. — merging

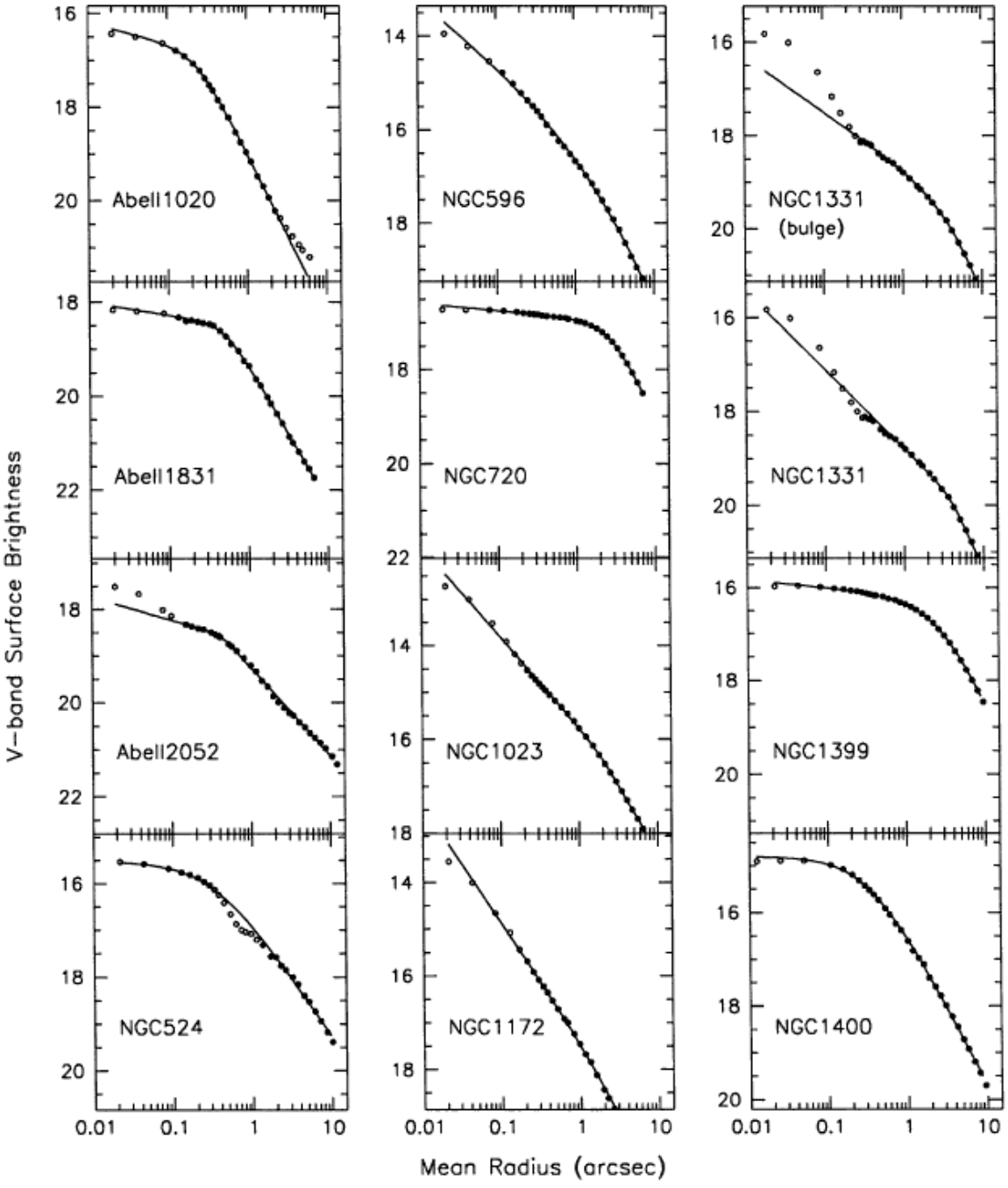
No theory predicts large cores.

Massive central BHs did not help

- Thermal evolution (Bahcall & Wolf 1976) forms $\rho \propto r^{-7/4}$
- Adiabatic growth of the central BH (Young 1980) forms $\rho \propto r^{-3/2}$

The HST view — Cores are not.

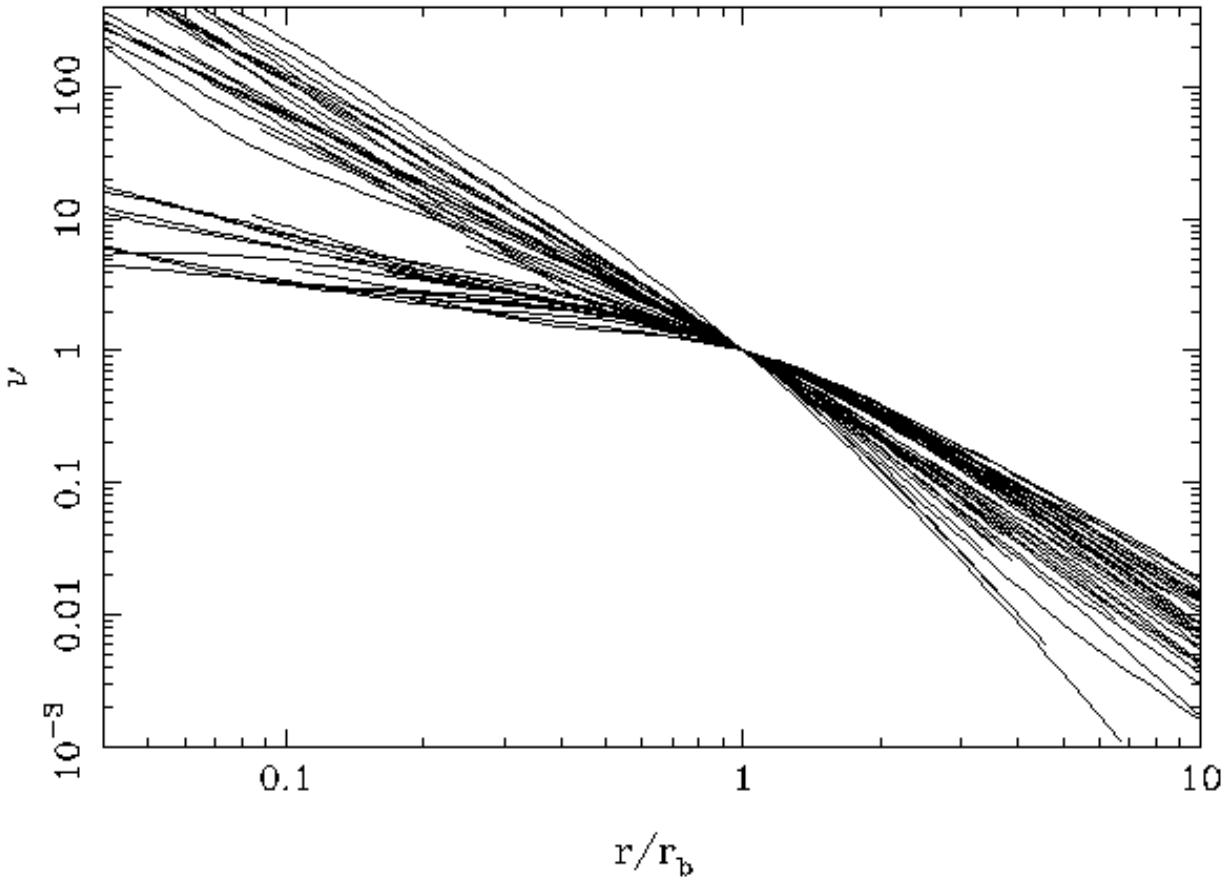
Byun et al. (1996)



No observed elliptical showed a real core...

Deprojected profiles.

Gebhardt et al (1996)



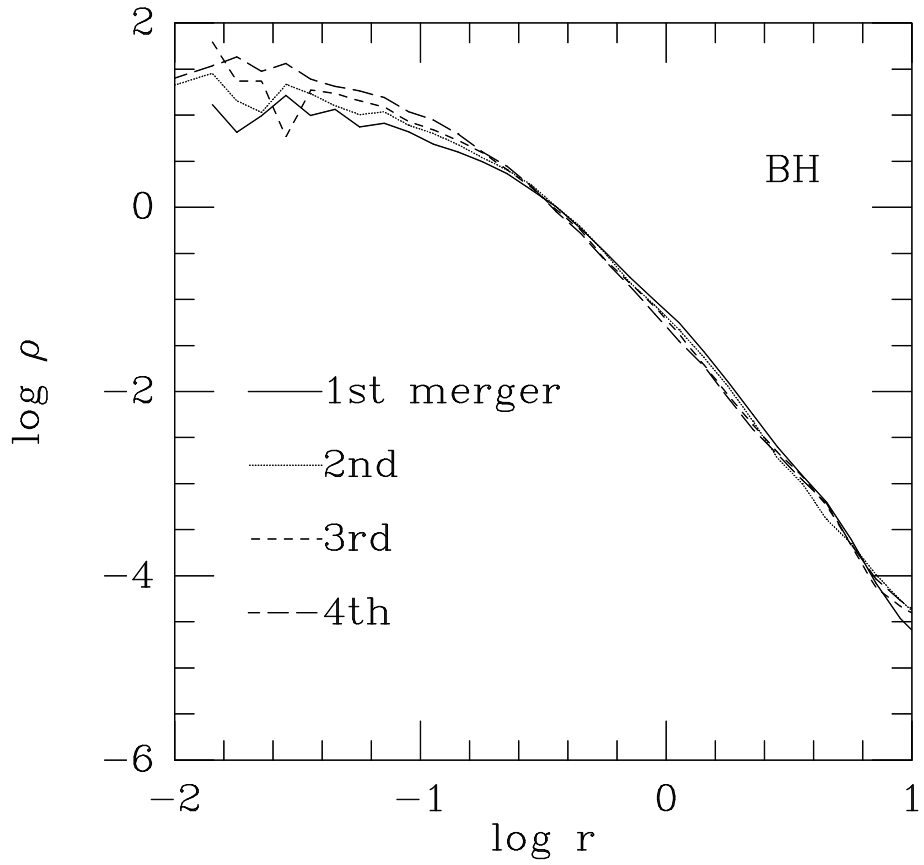
- Cuspy galaxies — $\rho \propto r^{-2}$
- Core (or "weak cusp") galaxies — $\rho \propto r^{-0.5\sim-1}$

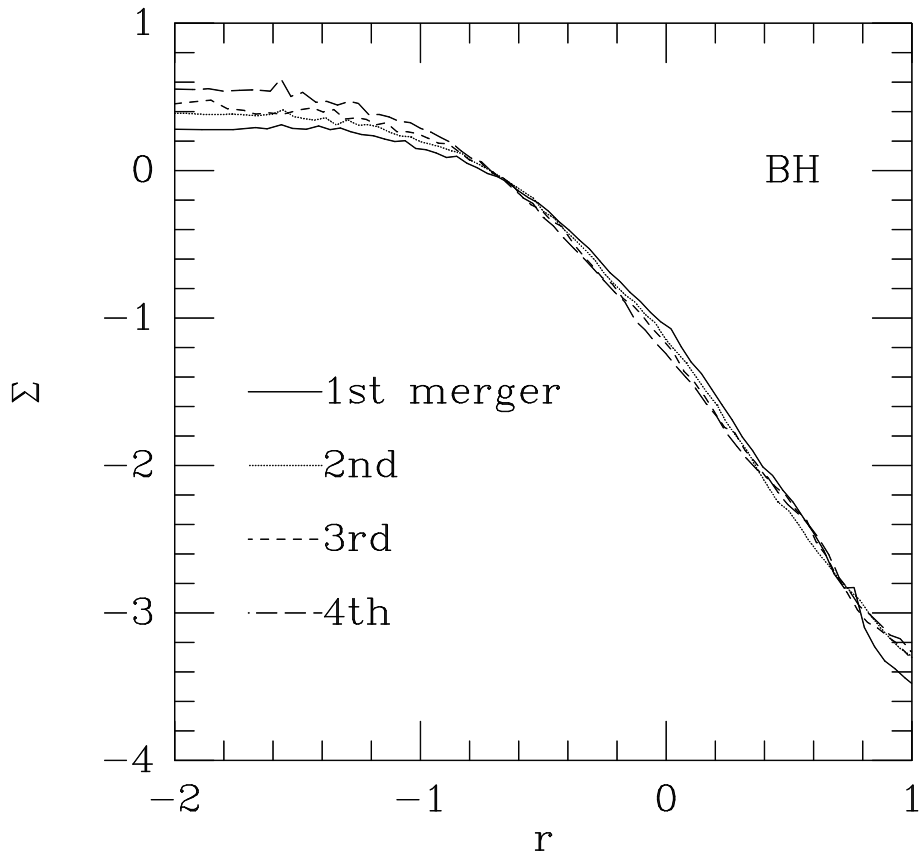
What have been recognized as "cores" are are all week cusps.

How these "weak cusp" ellipticals are formed?

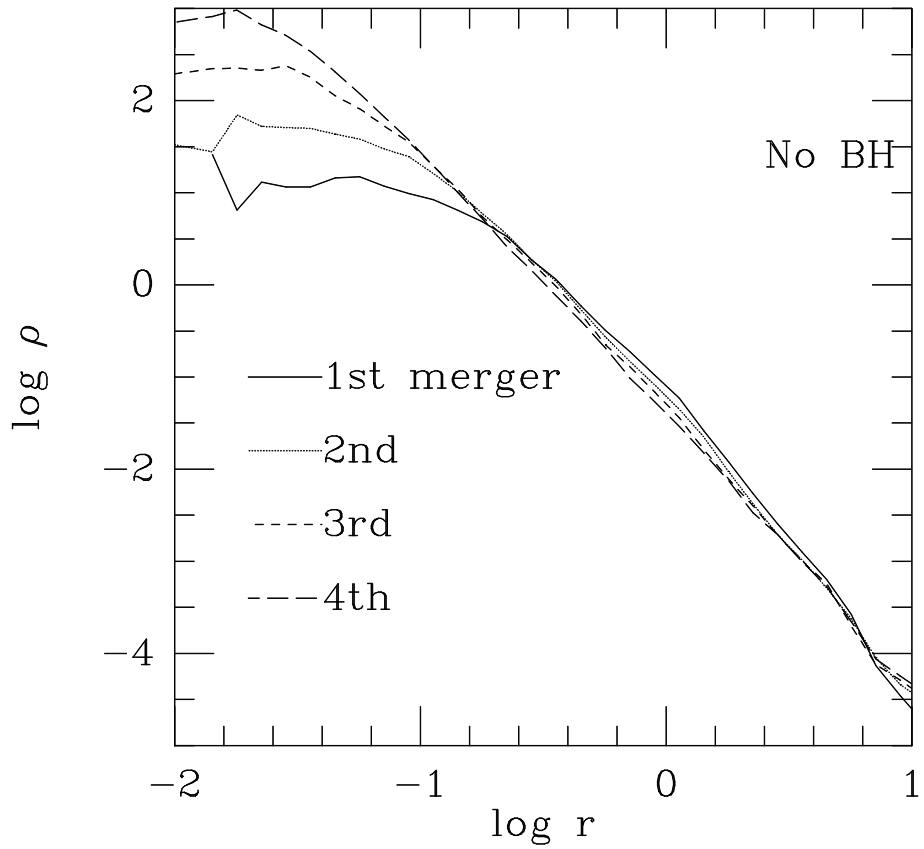
Merging with central BH

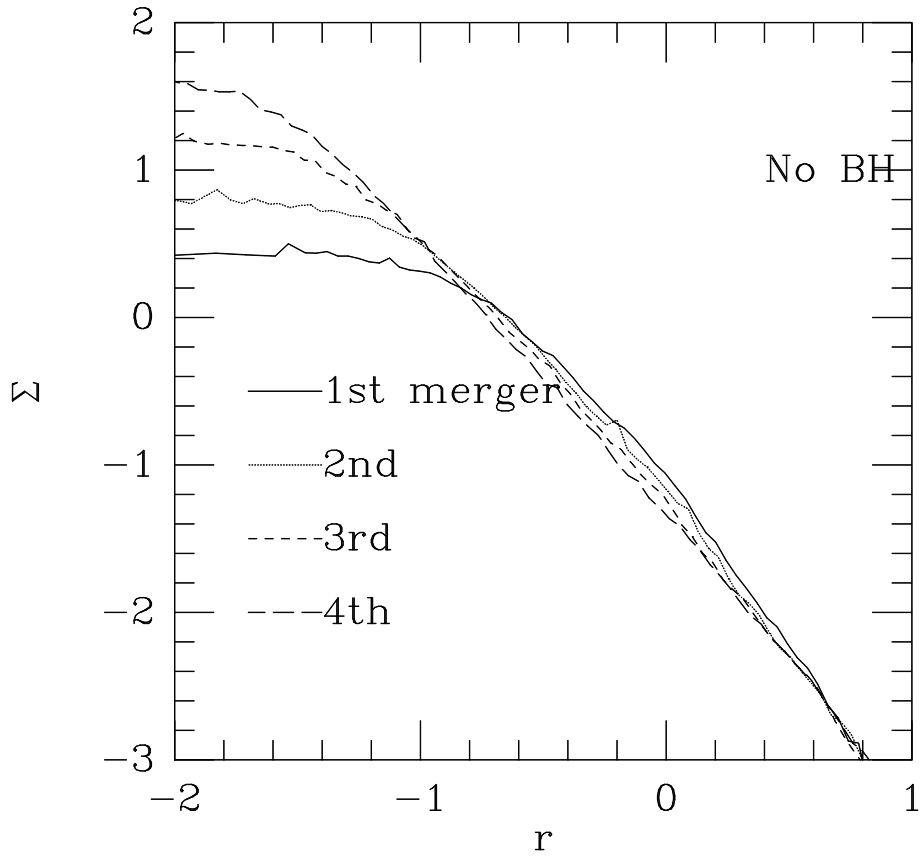
Makino & Ebisuzaki (1996)





Without central BH...





Simulation result summary

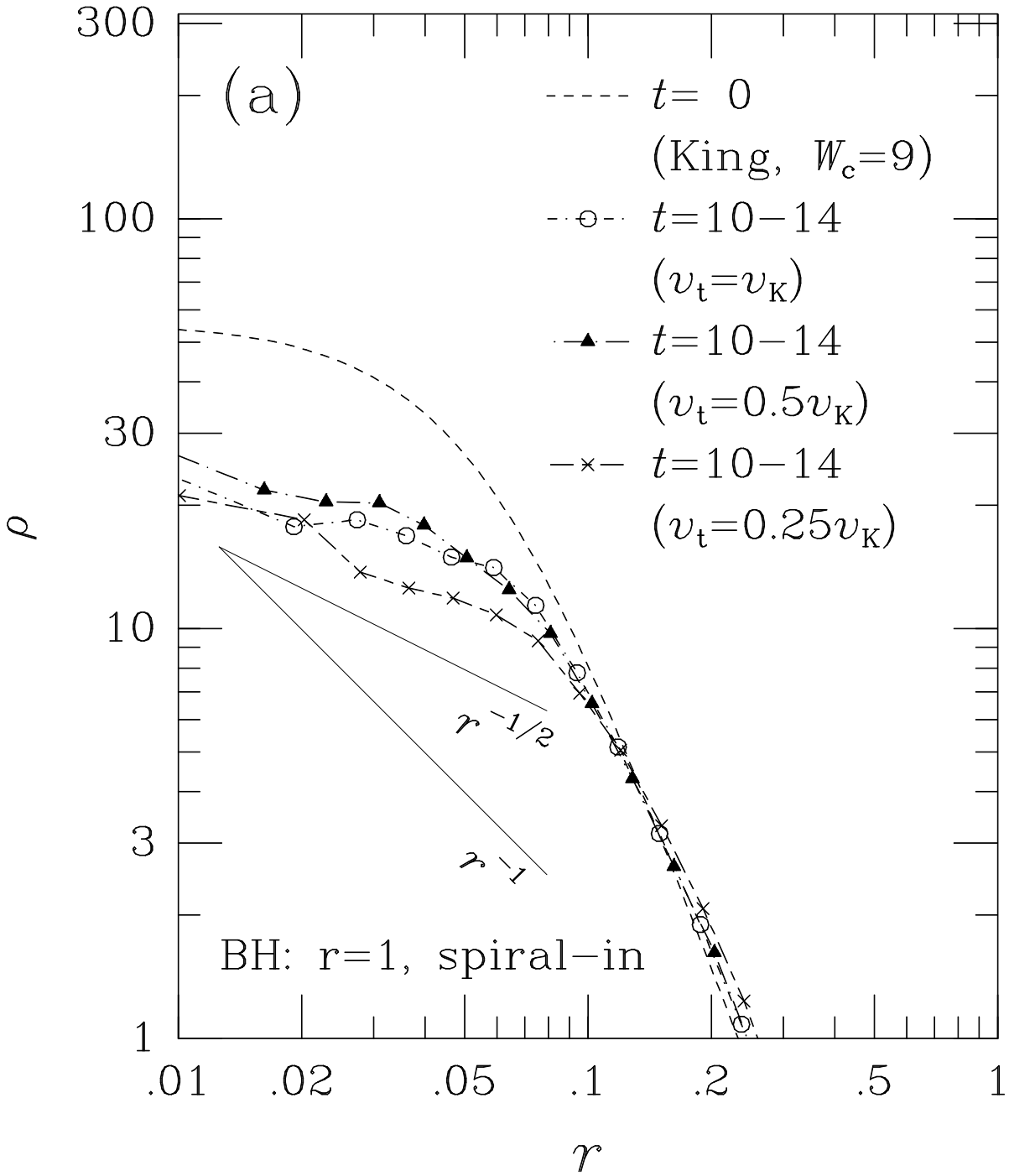
- Merging of galaxies with central BHs results in "weak cusps"
- $r_c/R_e \sim M_{BH}/M_g$

Seems to explain observation rather well.

What is the formation mechanism of "weak cusps"?

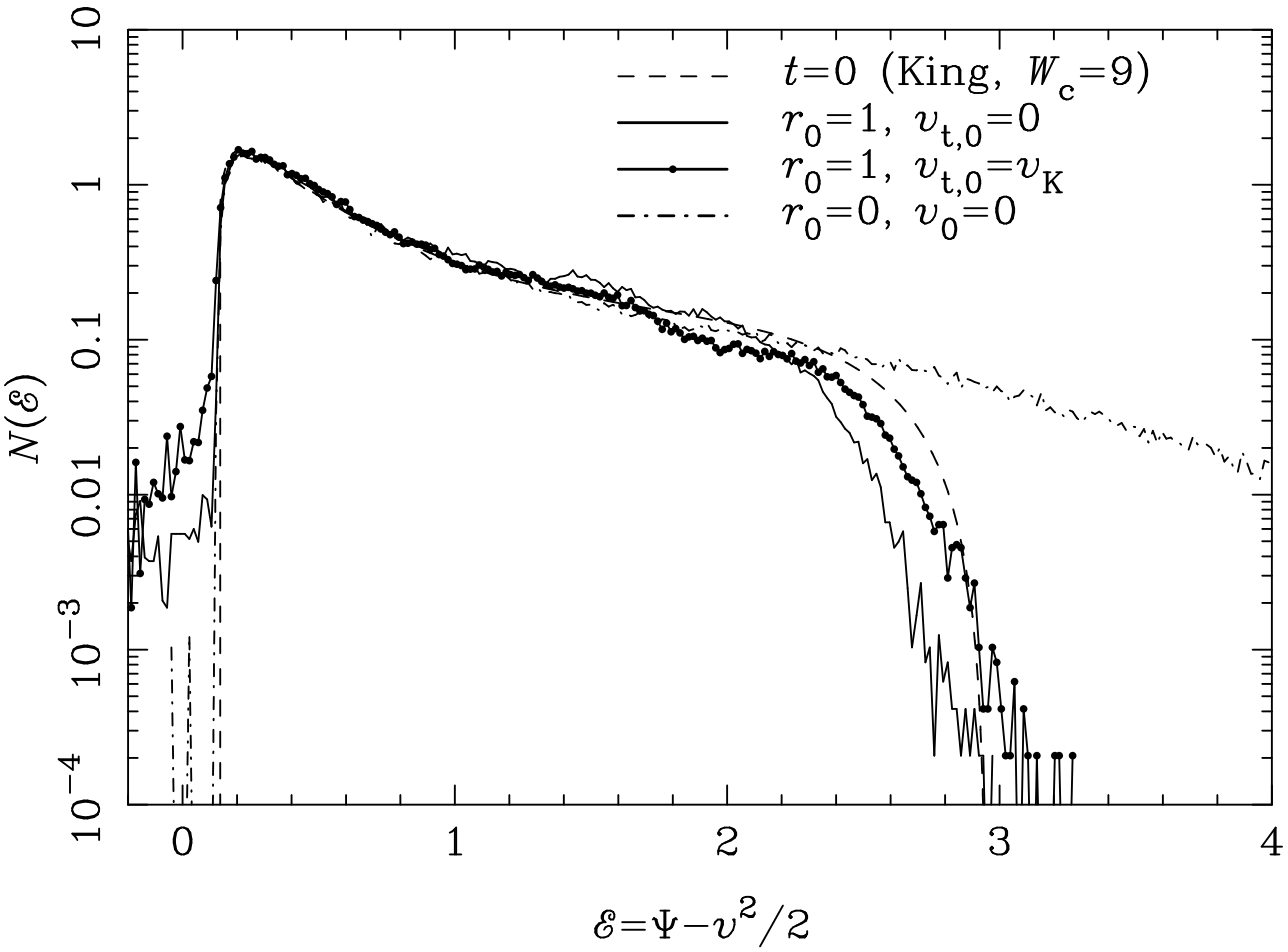
Simple Experiment

Let a BH fall down to the center of a spherical galaxy (Nakano and Makino 1999)



The same weak cusp was formed, but WHY?

Energy distribution function



Energy distribution function is practically unchanged

↓

$N(\mathcal{E})$ becomes zero at finite $\mathcal{E}=\mathcal{E}_0$.

No star is strongly bound to the central BH.

↓

$f(\mathcal{E})(=N(\mathcal{E})/A(\mathcal{E}))$ also becomes zero at finite E.

Theory for the weak cusp

$f(\mathcal{E}) \rightarrow \rho(r)$

$$\rho(r) = 4\pi \int_0^{\Psi(r)} f(\mathcal{E}) \sqrt{2\left[\Psi(r) - \mathcal{E}\right]} d\mathcal{E},$$

If $\Psi(r) > \mathcal{E}_0$, the above can be rewritten as

$$\rho(r) = 4\pi \int_0^{\mathcal{E}_0} f(\mathcal{E}) \sqrt{2\left[\Psi(r) - \mathcal{E}\right]} d\mathcal{E}.$$

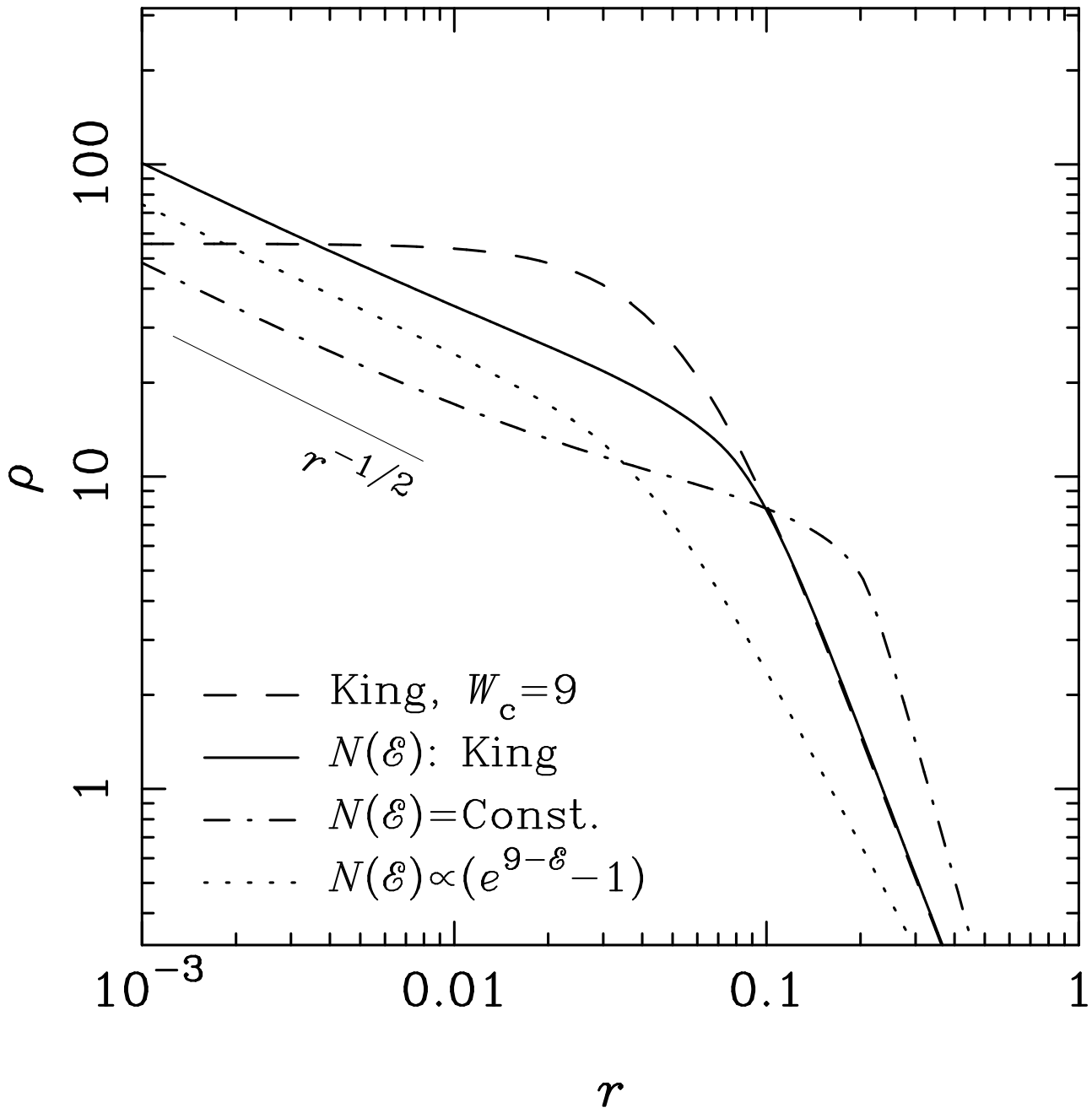
In the region $\Psi(r) > \mathcal{E}_0$, the right hand side can be expanded as

$$\rho(r) = 4\sqrt{2}\pi \sqrt{\Psi(r)} \int_0^{\mathcal{E}_0} f(\mathcal{E}) \left[1 - \frac{1}{2}\frac{\mathcal{E}}{\Psi(r)} + O\left(\left[\frac{\mathcal{E}}{\Psi(r)}\right]^2\right)\right] d\mathcal{E}$$

$$\rho(r) \propto \sqrt{\Psi(r)} \sim \sqrt{\frac{GM_{\rm BH}}{r}}.$$

Numerical example

- A self-consistent density profile for given $N(\mathcal{E})$.
- Gravitational field: self-gravity + central BH.
- Algorithm: Binney (1982)



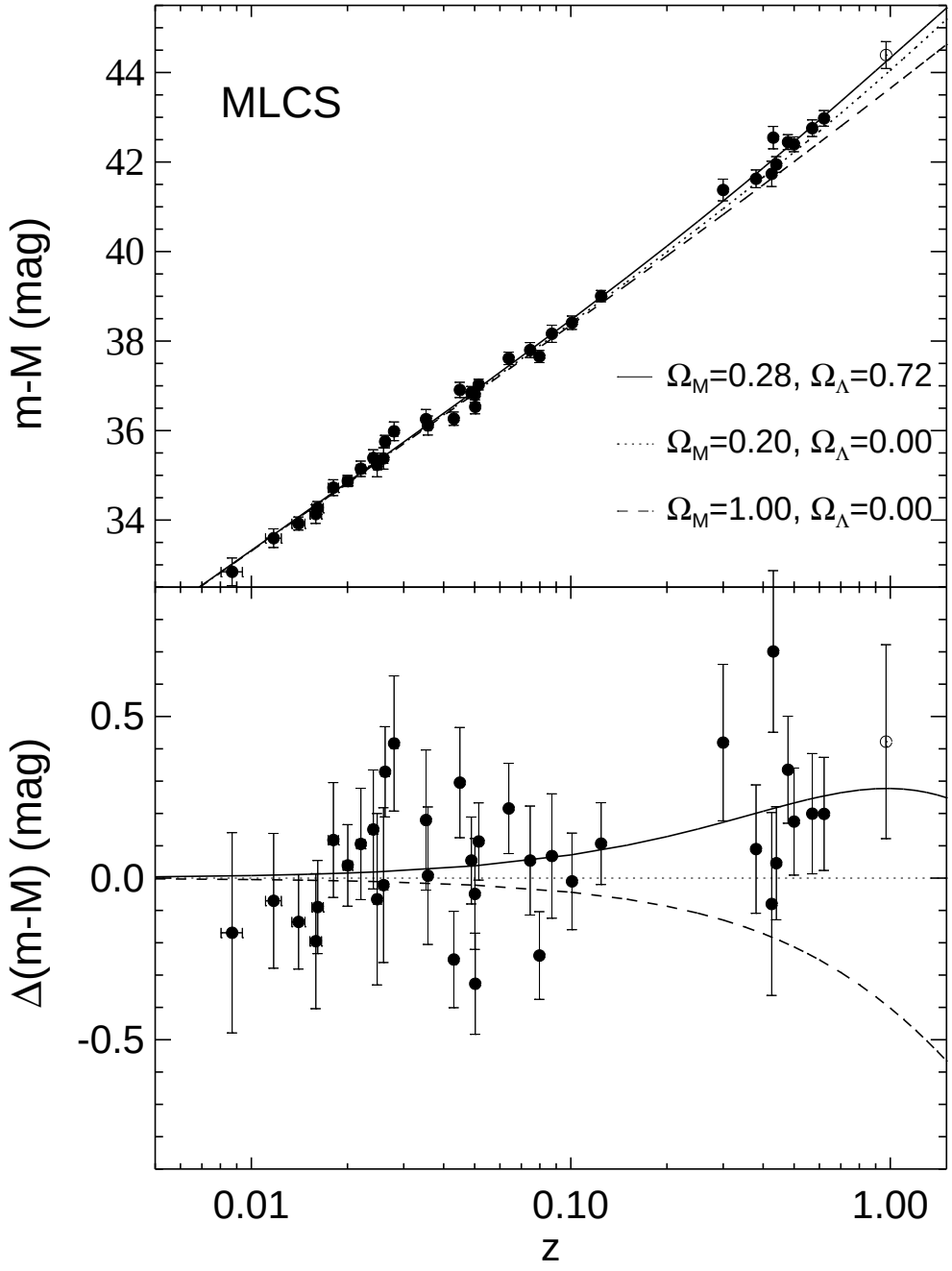
Summary for Ellipticals

- We found the weak cusp is formed in merging of galaxies with massive BHs.
- The formation mechanism of the weak cusp is explained by simple analytical argument.
- Weak cusps are signatures of the merging of galaxies with BHs

Distant SNe: What's the problem?

The resent results from High-z Supernova Search (astro-ph/9905049, also see 9805201, 9805200) and Supernova Cosmology Project (astro-ph/9812133) "proved":

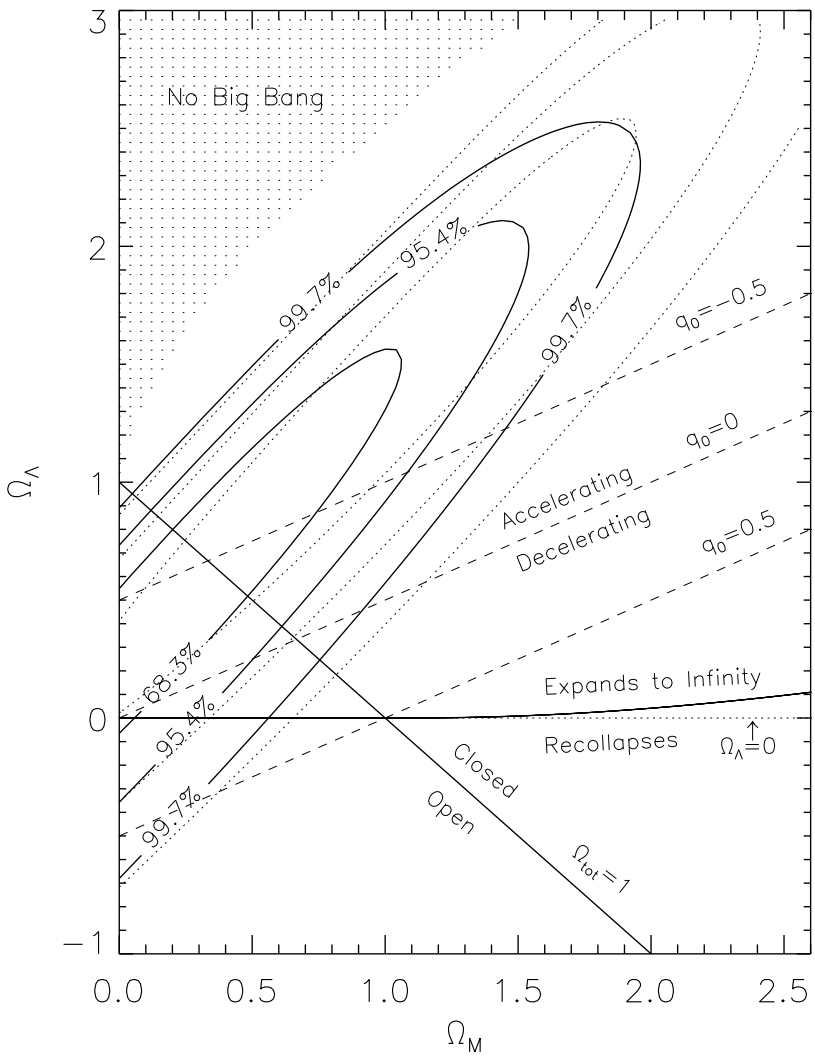
NON-ZERO Λ is necessary to explain the observed luminosity of High-z SNe



Possible sources of bias

- **Evolution**
- **Extinction**
- **Weak gravitational lensing**
- **...**

They argue everything is under control...
Here we discuss the weak lensing.



Weak lensing

Weak lensing by

- Large scale structure
- Clusters
- Galaxies
- individual stars

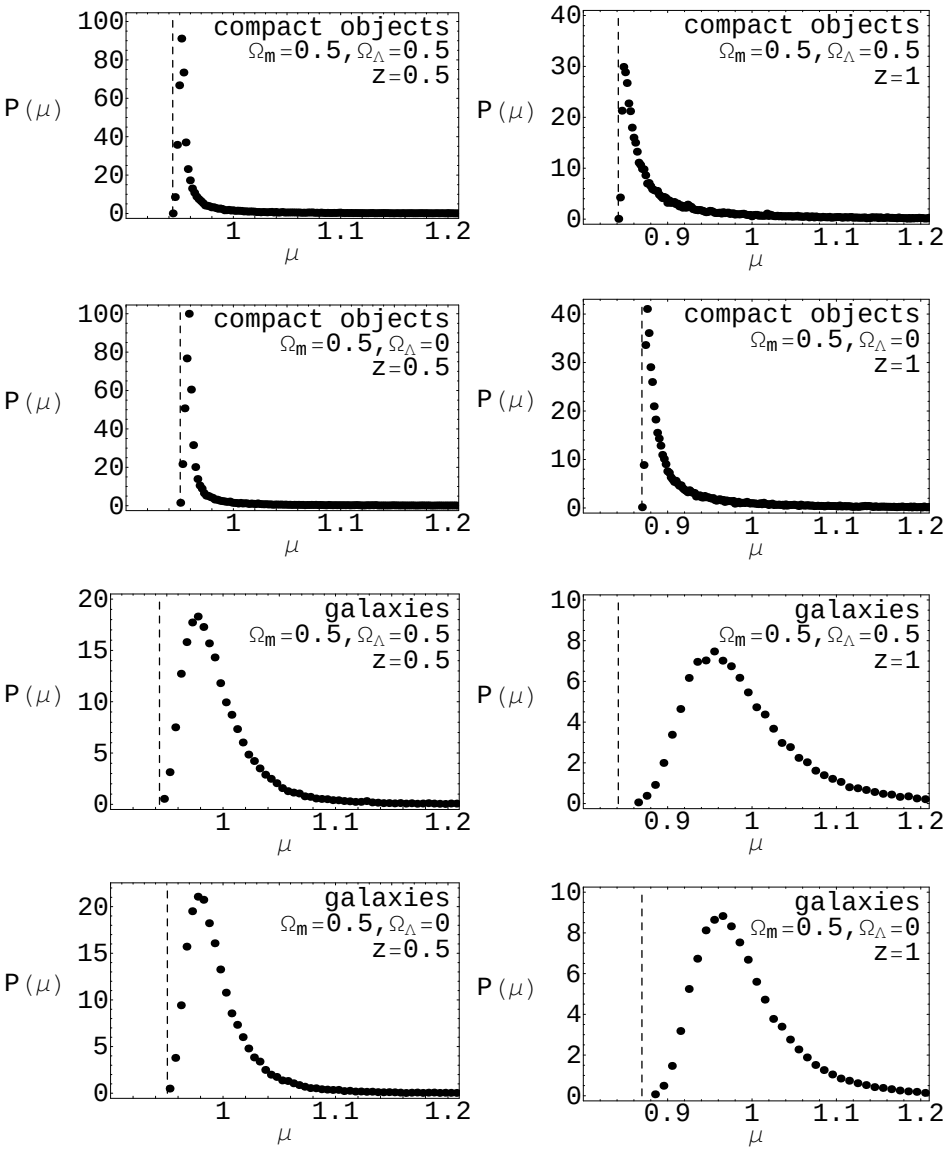
may reduce *median* luminosity of distant SNe

Numerical simulations

- Large scale structures have negligible effect (Wambsganss et al 1998)
- Galaxies have negligible effect (Holz 1998)

They performed "ray-tracing" through simulated large-scale structure or random distribution of model galaxies.

Calculation by Holz(1998)



- If galaxies were point-mass particles, the effect would be LARGE.
- Galaxies of REALISTIC SIZE have much smaller effect.

Galaxy model in Holz (1998)

What Holz considered a real galaxy is a truncated isothermal sphere with

- mass of $8.5 \times 10^{12} M_{\odot}$
- truncation radius of 380kpc (for $\Omega_m = 0.5$)

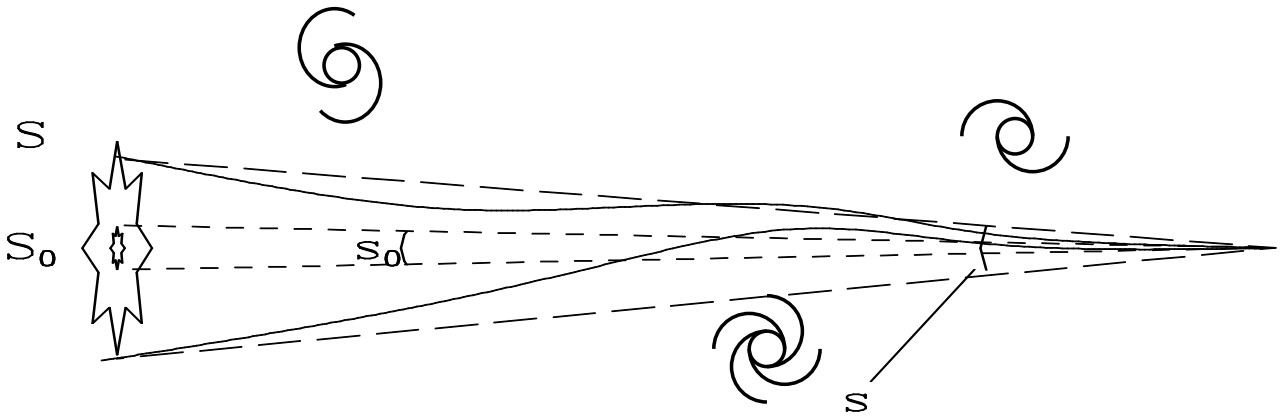
One might ask...

— Aren't they, well, a little bit *too large?*

— What is the effect of changing the size and mass of galaxies?

We studied the second question in a different context (Fukushige et al 1994a, 1994b, 1995; Funato and Makino 1994)

Exponential Divergence — Mechanism of Weak Lensing



Rays "emitted" from the observer are scattered by lenses.

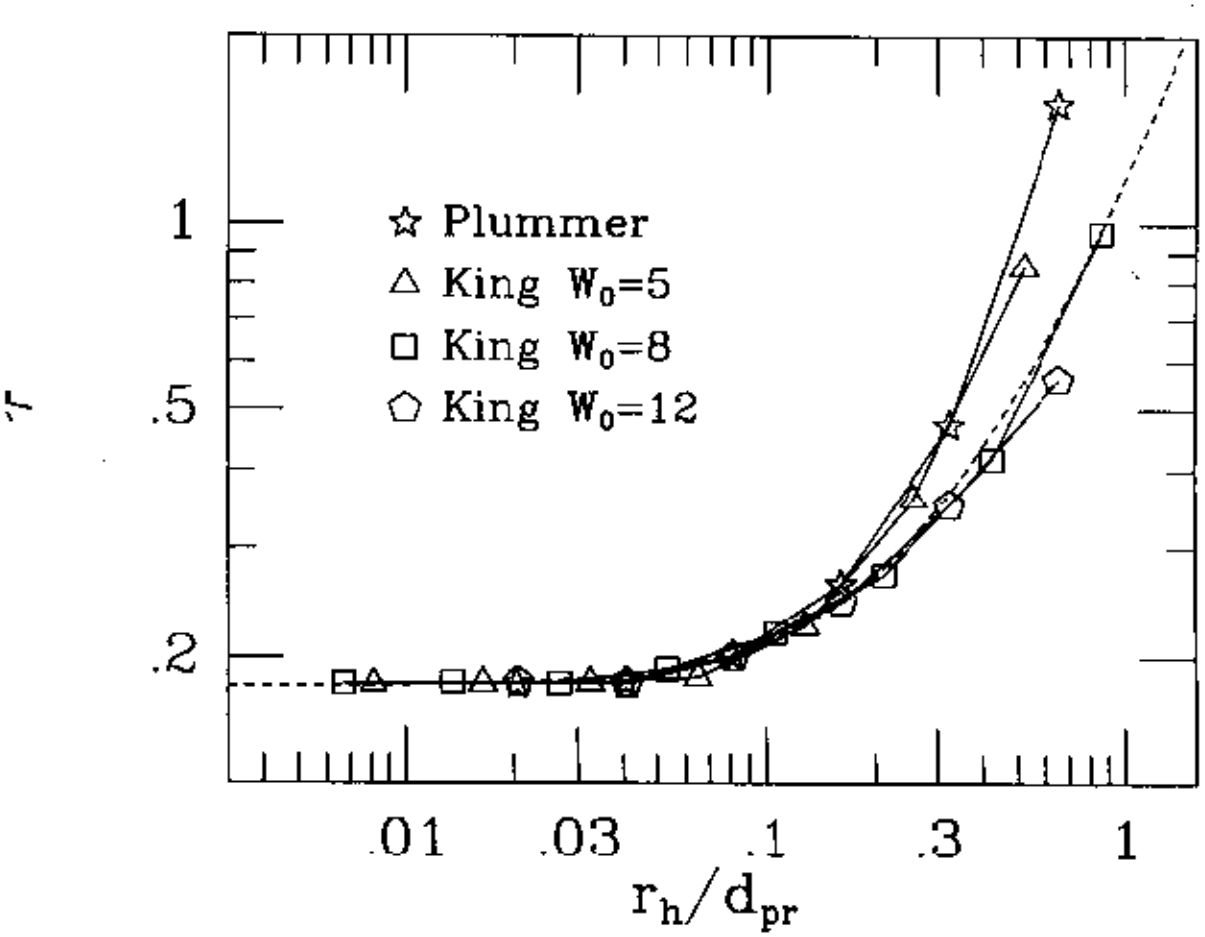
Cumulative effect of multiple scattering is multiplicative, not additive.

→ resulting in an exponential demagnification (power law of z for a flat universe).

It is necessary that the galaxies are "small", so that the rays go between galaxies.

How small is "small"?

Fukushige et al (1995, PASJ)
Timescale of the exponential growth



If $r_h < d_{pr}$, galaxies are "small".

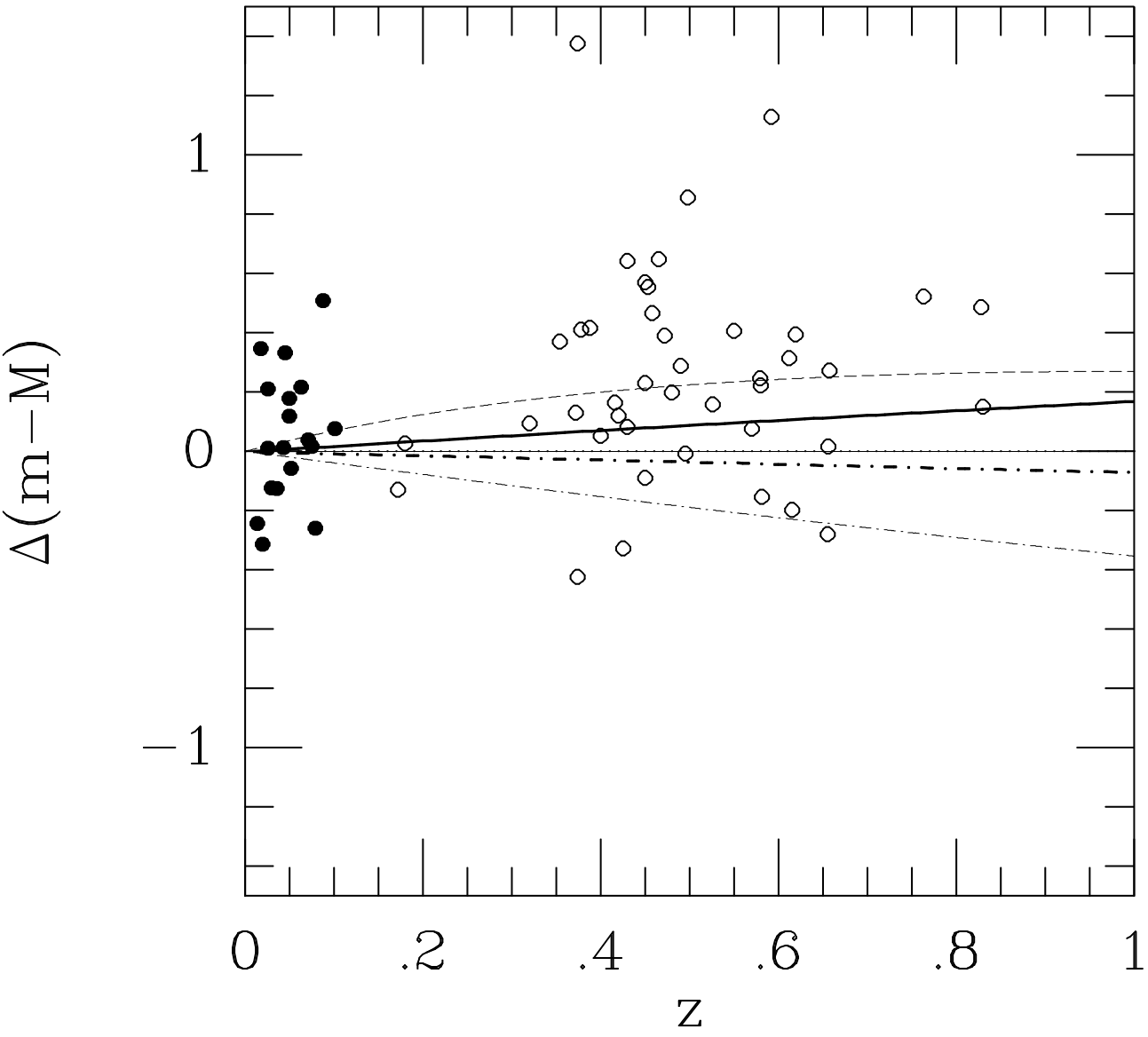
$$d_{pr} = \left(\frac{Gm}{c^2 n}\right)^{1/4} \sim 170\mathrm{kpc}$$

(For $M_g = 10^{12} M_\odot$, $\Omega_g = 0.2$, $H = 100\mathrm{km/Mpc}$)

Galaxies are much smaller than d_{pr}.

Effect of week lensing

Preliminary result from Funato and Makino (1999)



- The effect of weak lensing is $\Delta M \sim 0.18$ at $z = 1$ ($\Omega_g = 0.28$)
- An $\Omega = 0.2 \sim 0.3$, $\Lambda = 0$ universe is not incompatible with SNe luminosities when weak lensing is correctly taken into account (in 3σ level).

Conclusion (for SNe part)

- Holz (1998) grossly underestimated the effect of weak lensing, because his assumptions on the mass and size of galaxies are too large
- With realistic assumptions on galaxy mass and size, the effect of weak lensing is $\Delta m \sim 0.25\sqrt{\Omega_g}$, large enough to explain the observed luminosities of high-z SNe without introducing Λ.
- If you prefer, you can fit the observation with a flat universe, such as $\Omega = 0.5, \Lambda = 0.5$, when the weak lensing is taken into account.