Stellar Dynamics of Massive Blackhole Binaries

Jun Makino
Center for Computational Astrophysics
and
Division Theoretical Astronomy
National Astronomical Observatory of Japan

Center for Computational Astrophysics

Talk structure

- **Brief overview**
  - **Interaction between an SMBH binary and surrounding stellar system**
    - * **Evolution of SMBH binary**
    - * **Effect on the stellar system**
- **Evolution of SMBH binary**
- **Effect on the stellar system**

Talk structure

- Brief overview
- Evolution of SMBH binary
  - The "simplest" case
  - Effects ignored in the simplified model
    - non-spherical parent galaxy
    - gas
    - massive perturber
    - unequal-mass binaries
- Effect on the stellar system
  - some early works
  - modern works

Interaction between an SMBH binary and surrounding stellar system

SMBH binary:

- **Formed by merger of two galaxies with SMBHs**
  - **Massive BHs sink toward the center and settle at the bottom of the potential well.**
  - **Two BHs eventually form a binary.**
- **"Hardens" in the way same as usual binaries in globular clusters**

Basic physics is the same as that of binaries in globular clusters

Behavior of binaries in globular clusters

- **Energy source: stops core collapse**
- **escapes as the result of recoil from binary-single star interaction. (Indirect heating)**
- **hardening rate: $dE/dt = \text{const.}$**

Differences

An SMBH binary is massive, in two different senses:

- **Mass scaled by the mass of the field stars is large — statistical argument breaks down**
- **Mass scaled by the mass of the parent stellar system is large — single SMBH binary can affect the structure of the parent galaxy**

Effect of large $M_{\mathrm{bh}}/M_{\mathrm{field}}$

- **Equipartition: SMBH binary has very small velocity (stays at the center of the galaxy)**
  - **Never escape from the center (except through triple BH interaction or gravitational wave recoil at merging)**
- **Large semi-major axis: Large loss-cone, slow refill.**
  - **Loss-cone depletion: SMBH binary can kick out all stars which can interact with the binary (have small enough angular momentum J).**
  - **Refill of loss-cone: timescale $\propto$ relaxation timescale.**

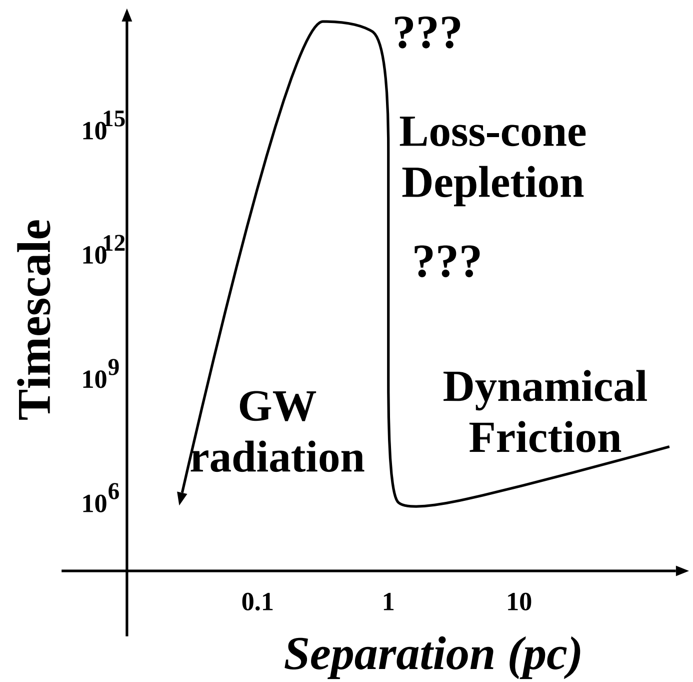
The classic picture

Begelman, Blandford and Rees (1980)

1. massive BH binary forms at the center of a merger remnant

2. It hardens to the point where the loss cone is depleted

3. After the depletion, evolution timescale $\propto$ (relaxation time)$^{-1}$



Massive black hole binary never merge

Last 20 years of numerical study of the stellar dynamics of SMBH binary

Central question: **Is the classic picture really true?**

- Simple case: equal-mass circular binary, spherical parent galaxy, pure single stellar system
- Reality: unequal-mass binary, initial eccentricity, triaxial galaxy, satellite galaxies/subhalos, gas

In short...

- **Simple case:**

  Binaries never merge

- **Other effects included**

  Binaries, in most cases, probably merge

Effect on the stellar system

Two aspects:

- **SMBH (binary or not) adds central potential**
- **SMBH binary kicks out nearby stars**

Classic theories of stellar system around massive BH

- **Adiabatic growth (Young 1980):**
  $\rho \propto r^{-3/2}$, $f(E) = \mathrm{const.}$ near the BH.
- **Thermal evolution (Bahcall and Wolf 1976):**
  $\rho \propto r^{-7/4}$, $f(E) = |E|^{1/4}$ near the BH.

Not applicable to evolution in the dynamical timescale..

Theory for changes in dynamical timescale

(I believe) the central cusp is pretty well understood. Nakano and JM(1999a, b)

$\rho \propto r^{-1/2}$, $f(E) = 0$ near the BH.

Detailed comparison with observations require numerical simulations.

Evolution of SMBH binary

- The “simplest” case
- Effects ignored in the simplified model
  - non-spherical parent galaxy
  - gas
  - massive perturber
  - unequal-mass binaries

The simplest case

- **equal-mass BHs**
- **near-circular initial orbit**
- **spherical galaxy with finite-density core (no cusp)**

Several years ago: *"numerical N-body experiments are not well suited to probe these mechanisms over long times due to spurious relaxation." (Milosavljvić and Merritt 2003)*

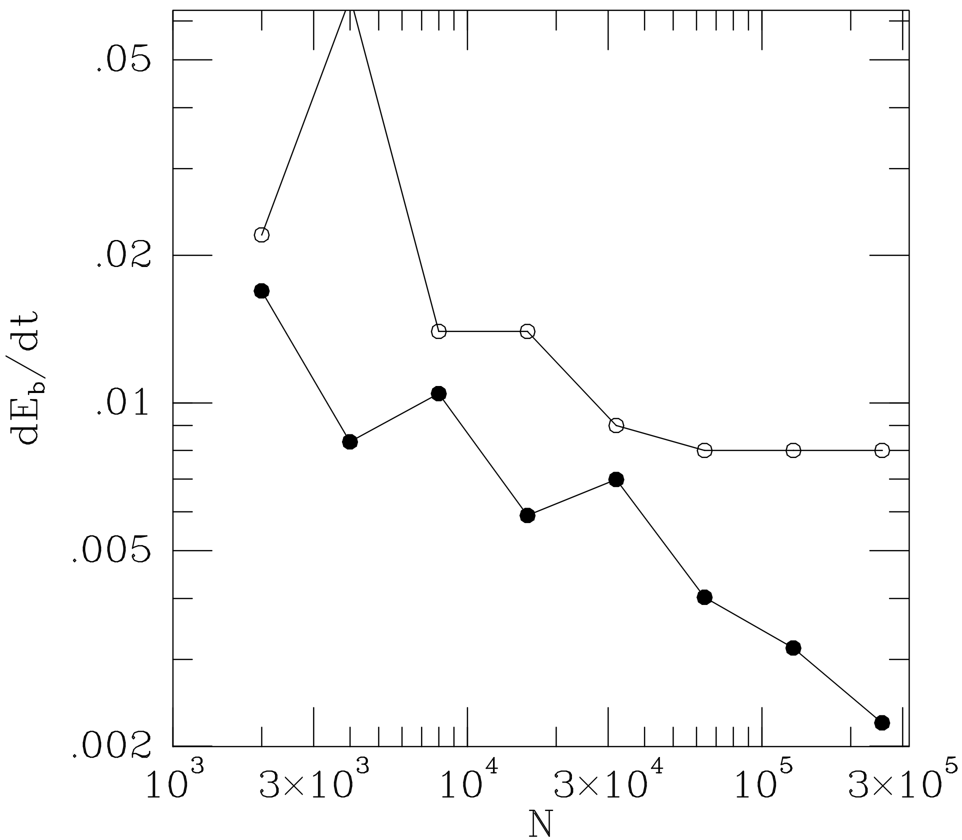
Numerical results available did not confirm the loss-cone depletion

Numerical works as of 2003

- JM 1997
- Quinlan 1997
- Milosavljević & Merritt 2001 (also 2003)
- Chatterjee, Hernquist & Loeb 2003

Results are not quite consistent with each other or with the loss-cone argument.

JM 1997 — Hardening rate



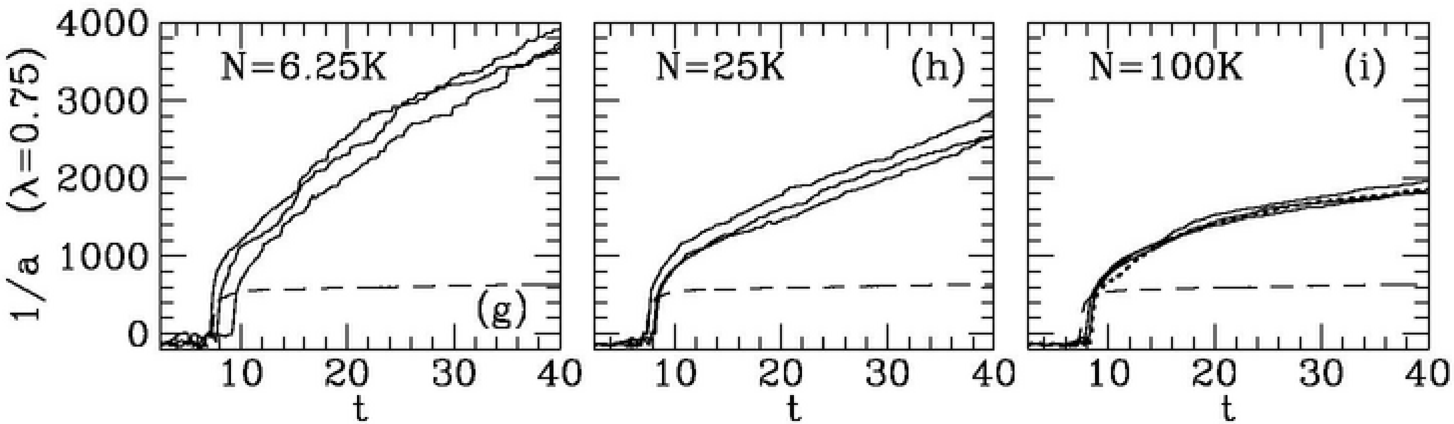
***N* up to 256K**

Upper: $E_b \sim 1/160$
Lower: $E_b \sim 1/10$

Late phase: Slope depends on *N*, but too weak (around $-1/3$)

Not consistent with thermal relaxation argument

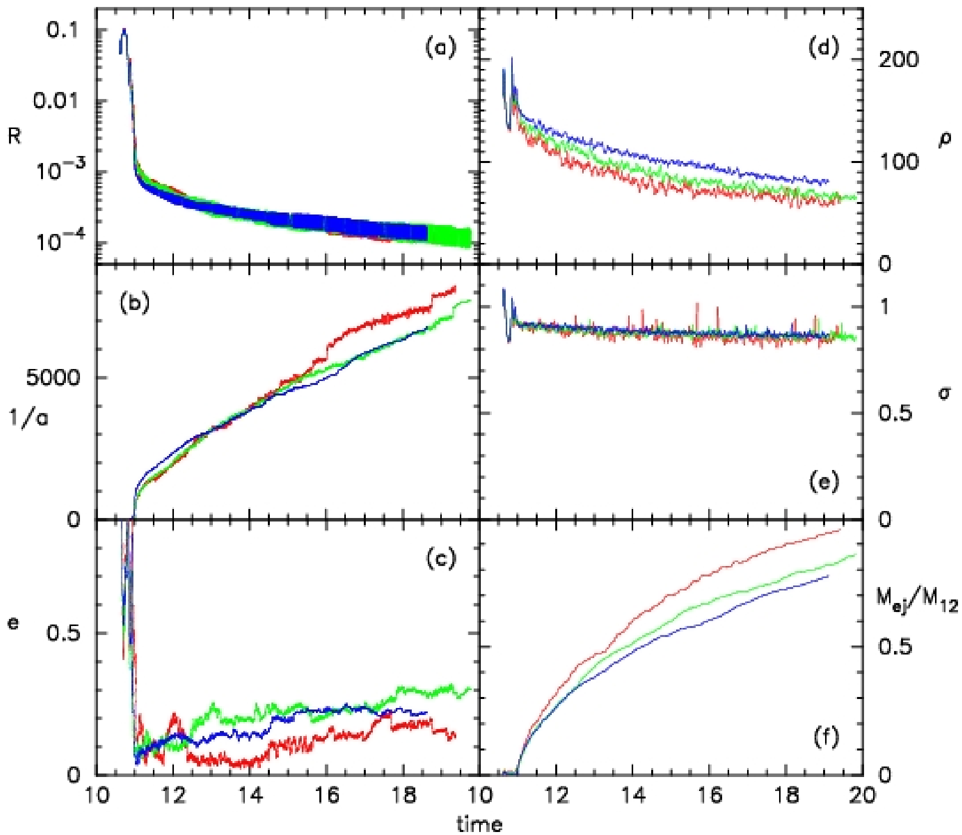
Quinlan 1997



N up to 200K (One of the curves in "N=100K" panel is for N=200K)

Hardening rate independent of N for $N > 100$K

Milosavljević & Merritt 2001



N **up to 32k**
Hardening rate independent of ***N***

Argued that they could not see ***N*** **dependence because** ***N*** **was too small (quite reasonable).**

Chatterjee, Hernquist & Loeb 2003

- **Same numerical method as Quinlan 1997**
- **N up to 400K**
- **Various M_{BH}**

Claim: Timescale independent of N for $N > 200K$ (I couldn't find any supporting figure or whatever in their paper though...)

The state of the art in 2003

No agreement at all...

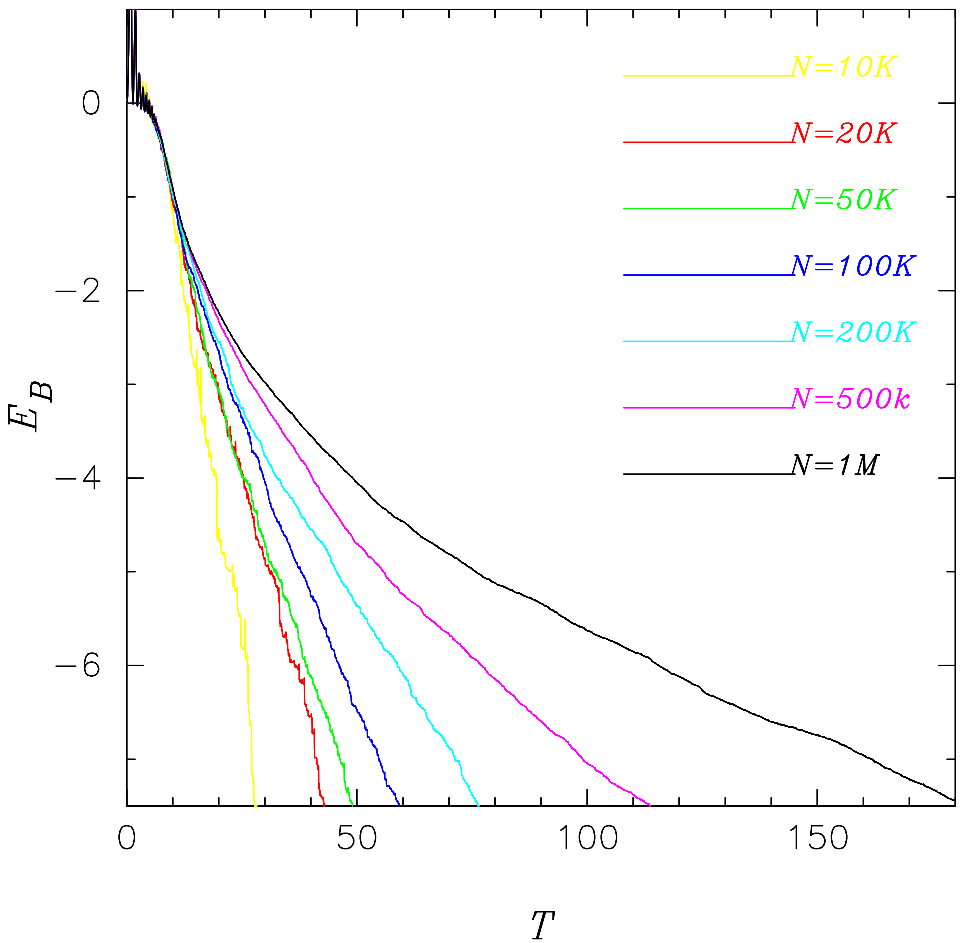
- $N \leq$ **256K**
- **No agreement between different people.**
- **No result consistent with the loss cone depletion argument.**

After 2004

Current situation is somewhat better:

- **JM and Funato 2004**
- **Berczik, Merritt, and Spurzem 2005**

JM and Funato 2004

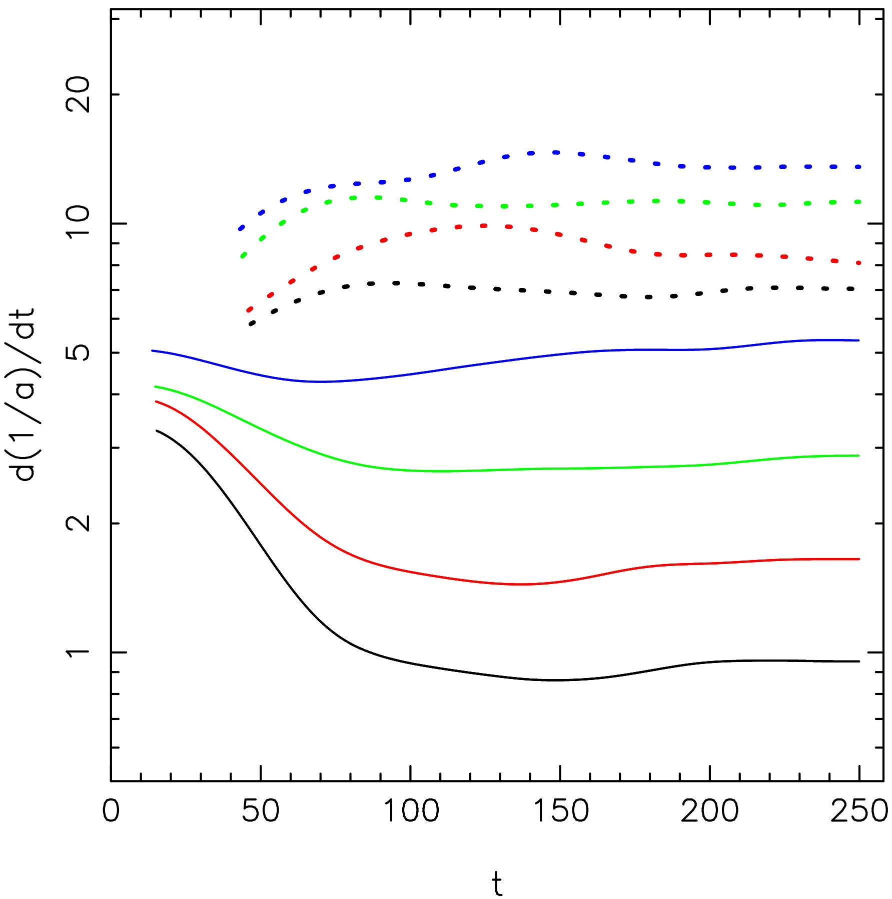


N up to 1M.
Hardening rate β depends on N.

If we write
$\beta \propto N^{-\gamma}$,
γ approaching to 1 for late phase

Not inconsistent with asymptotic value being 1.

Berczik et al 2005



N up to 0.4M
Simulation significantly longer than JM and F 2004.

N dependence
$\sim N^{0.8}$ ($M_{bh} = 0.02$)
$\sim N^{0.33}$??
($M_{bh} = 0.005$)

Summary of BHB N-body simulations

- N much larger than old simulations
- Duration also longer
- Growth rate shows clear dependence on N
- Results not converged yet...
- SMBH binaries do not merge in Hubble time.

Effects ignored in the simplified model

- Gas (Thursday and Friday)
- Non-spherical galaxies (Berczik et al. 2006)
- Non-equal-mass BHs (Matsubayashi et al. 2007, Iwasawa (this conference))
- Triples (Iwasawa et al. 2006)

Large mass ratio

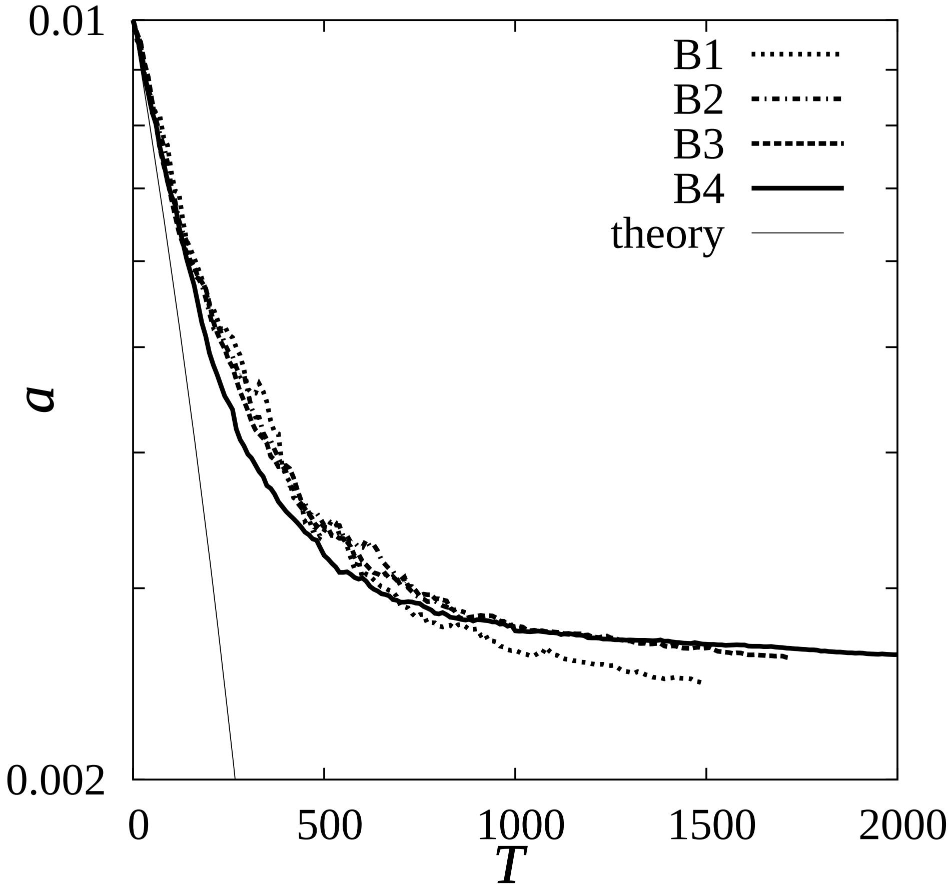
- **Equal-mass SMBH binary does not merge.**
- **Clearly, equal mass is a very special case.**

Mergers of parent galaxy: dominated by minor mergers. Typical mass ration is 1:10. IMBH-SMBH mergers, if ever occur, have even larger mass ratios.

Model calculation (Matsubayashi et al 2007)

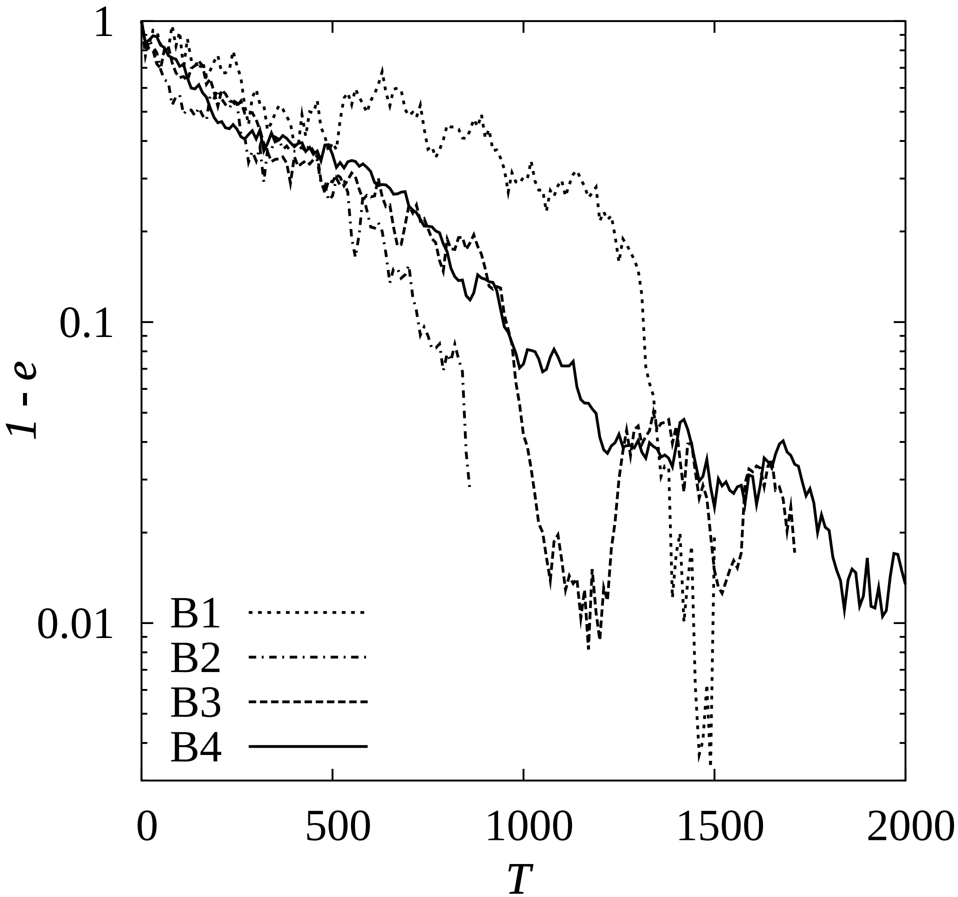
- **Initial condition: Bahcall-Wolf**
- **SMBH $3 \times 10^6 M_\odot$**
- **IMBH $3 \times 10^3 M_\odot$**
- **System of units: 1pc, 4600years**
- **Mass of stars: $3M_\odot$ (highest resolution case)**

Orbital evolution



IMBH placed at 0.01pc
Evolution slows down (loss cone depletion)

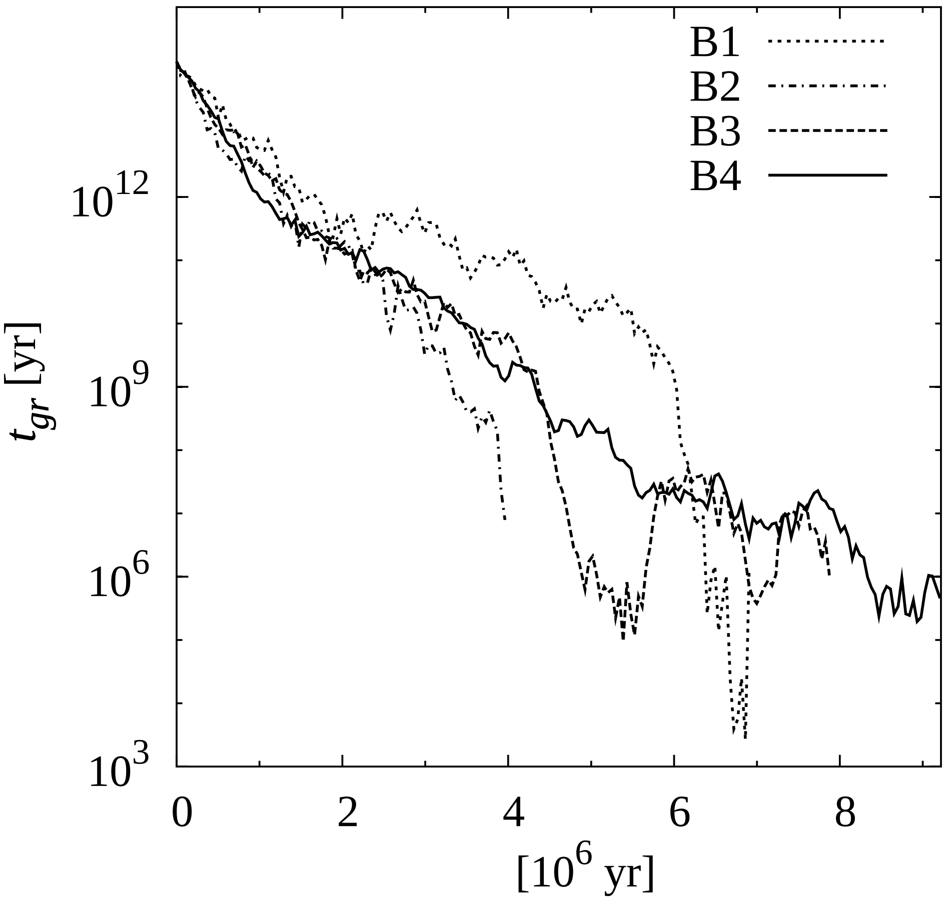
Eccentricity



Eccentricity goes up

Not observed in equal-mass case

GW timescale



Becomes very short because of high *e*.

SMBH and IMBH easily merge.

Why eccentricity goes up?

Wait for Masaki's talk.
The essence:

- **potential around SMBH+IMBH: pure Kepler + small perturbation**
- **If IMBH has non-zero eccentricity, perturbation potential is not axisymmetric.**
- **Angular momenta of stars are not conserved. Their orbits become chaotic.**
- **This means the loss-cone refill timescale is not relaxation timescale, but the timescale of orbital change due to IMBH perturbation. (Similar to Kozai cycle)**

Why eccentricity goes up? (cont'd)

- When stars interact with large-mass-ratio binary, stars in prograde orbits are ejected more efficiently than those in retrograde orbits.
- Therefore, on average escaped stars carry away angular momentum from SMBH-IMBH binary. Angular momentum is taken out of the parent stellar system, but chaotic change of orbits make the parent stellar system non-rotating. The back reaction to the IMBH makes it eccentric.

Might explain high eccentricity of observed systems (OJ287, Valtonen et al 2008 etc)

Effect on the stellar system

Two aspects:

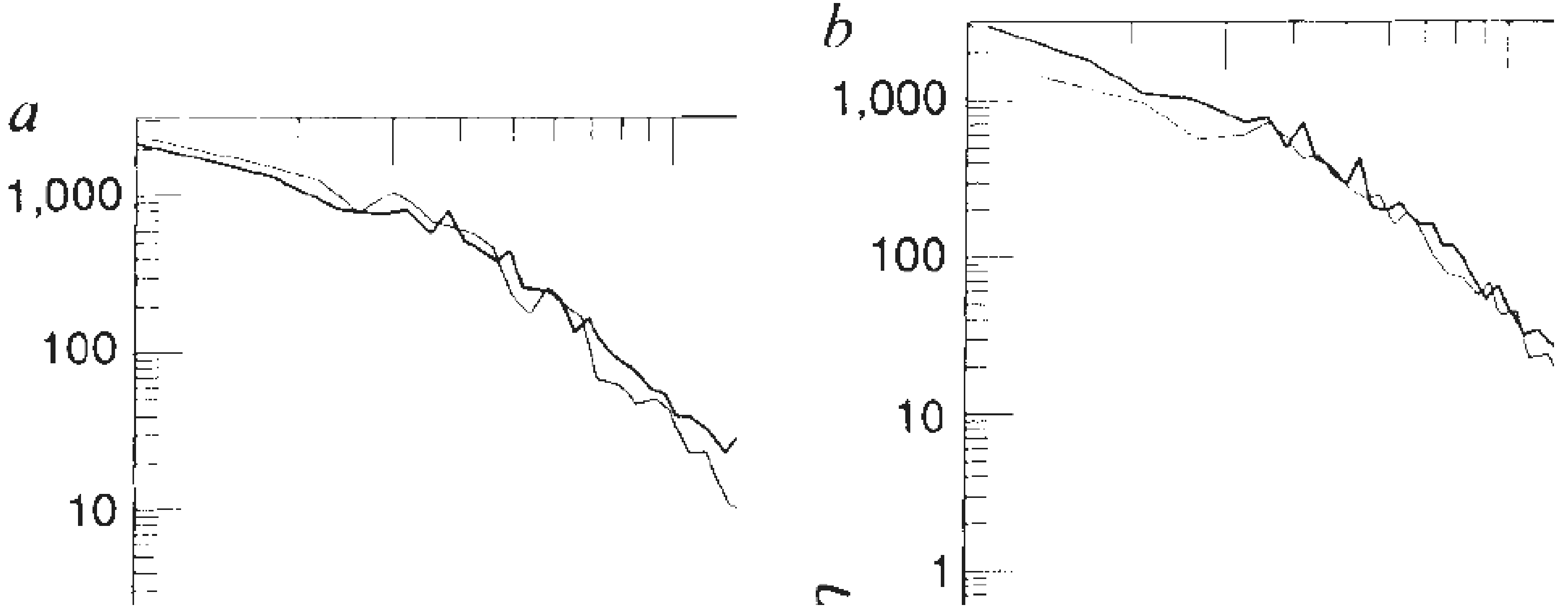
- **SMBH (binary or not) adds central potential**
- **SMBH binary kicks out nearby stars**

(Personal) Historical view

To my knowledge, Ebisuzaki, JM and Okumura (1991) seems to be the first paper to discuss:

- **Effect of SMBH binary to the structure of its parent galaxy**
- **Co-evolution of SMBH and galaxy...**

EMO 1991 result



With central BHs **Without central BHs**

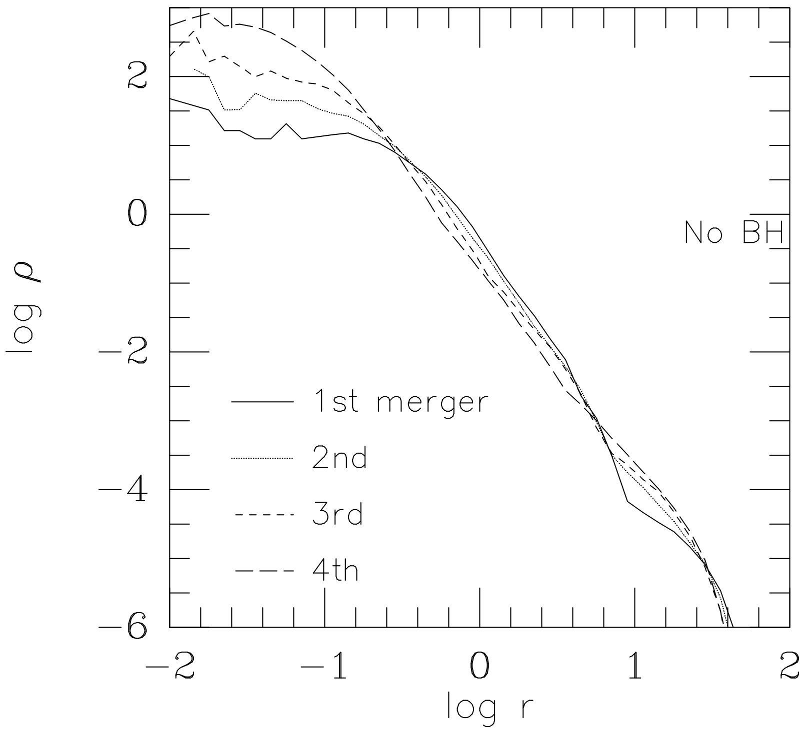
Thick: merger remnant, thin: progenitor

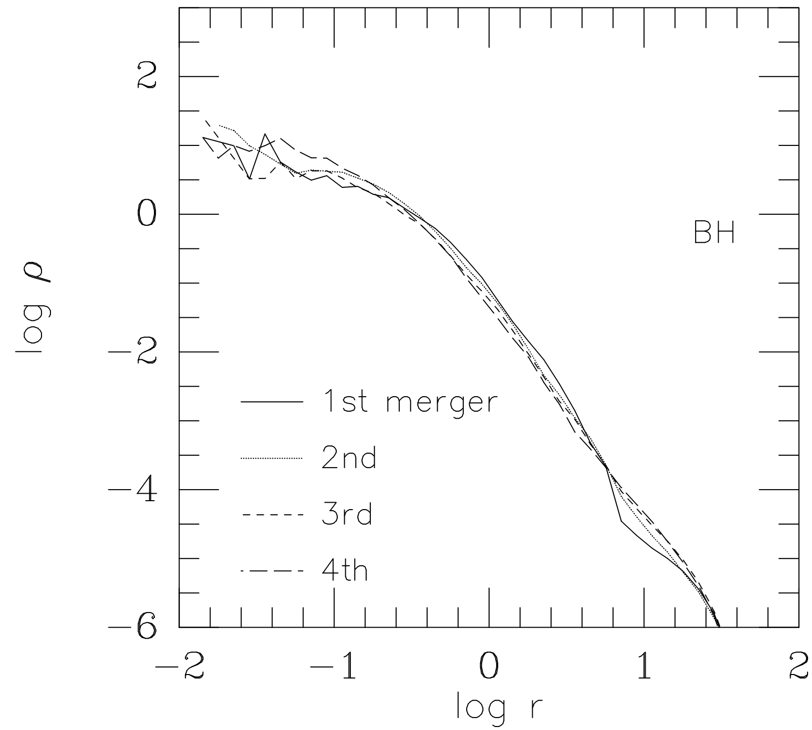
If you look *very carefully*, you can see

- **central density decreases for run with BH**
- **cent-ran density increases for run without BH**

SMBH binary can let the central region expand

Makino 1996





With central BHs **Without central BHs**

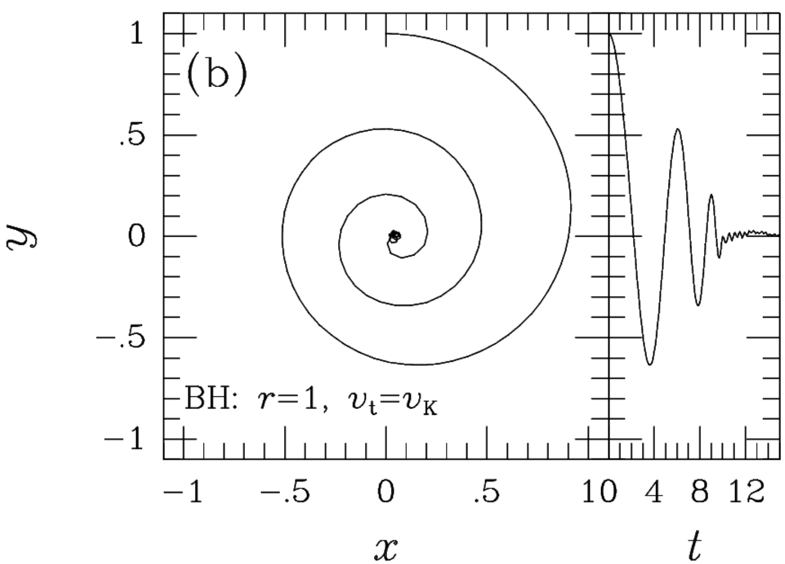
Repeated merger, mass & size scaled after each merger

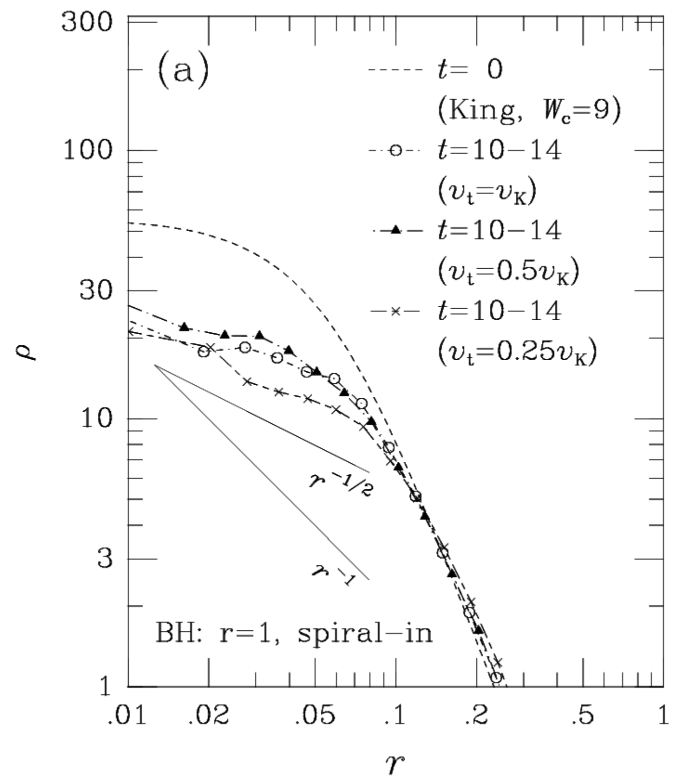
With BH, the structure of parent galaxy converges to a single profile, determined by M_{BH}/M

Central structure

Nakano and Makino (1999a, b)

You do not need two BHs to change the central structure





Central structure (cont'd)

Assume: $f(E) < 0$ for $E < E_0$

$$
\begin{aligned}
\rho(r) &= 4\pi \int_{E_0}^{0} f(E)\sqrt{2\left[E-\phi(r)\right]}dE \\
&= 4\sqrt{-2\pi\phi(r)} \int_{E_0}^{0} f(E)\left[1-\frac{1}{2}\frac{E}{\phi(r)}+O\left(\left[\frac{E}{\phi(r)}\right]^2\right)\right]dE \\
&\sim 4\sqrt{-2\pi\phi(r)} \int_{E_0}^{0} f(E)dE \\
&\sim \sqrt{\frac{GM_{\mathrm{BH}}}{r}} \int_{E_0}^{0} f(E)dE.
\end{aligned}
$$

$$\rho \propto r^{-1/2}.$$

The mechanism which determines the size of the cusp region is, in my opinion, not well understood.

Near future of N-body simulation

- **New computer**
- **New high-order integration scheme**
- **New tree-direct hybrid scheme**

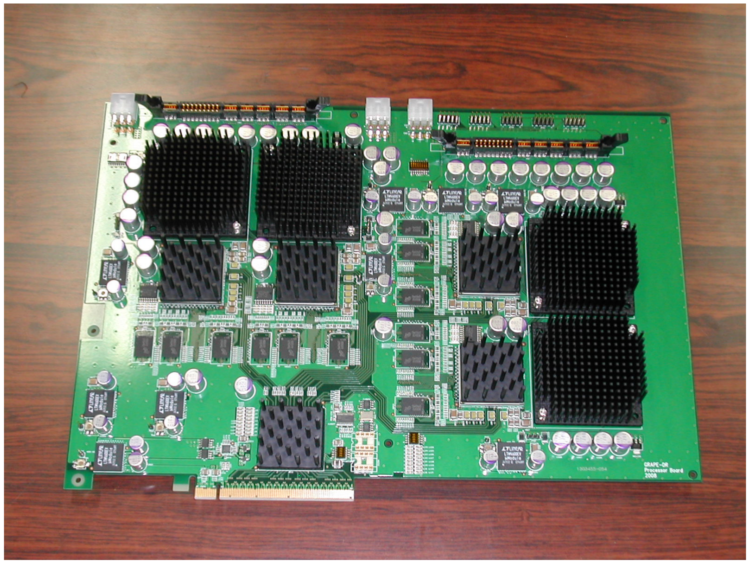
New computer — GRAPE-DR

- **Planned peak speed: 2 Pflops**
- **New architecture — wider application range than previous GRAPEs**
- **primarily to get funded**
- **No force pipeline. SIMD programmable processor**

The Chip

Sample chip delivered May 2006

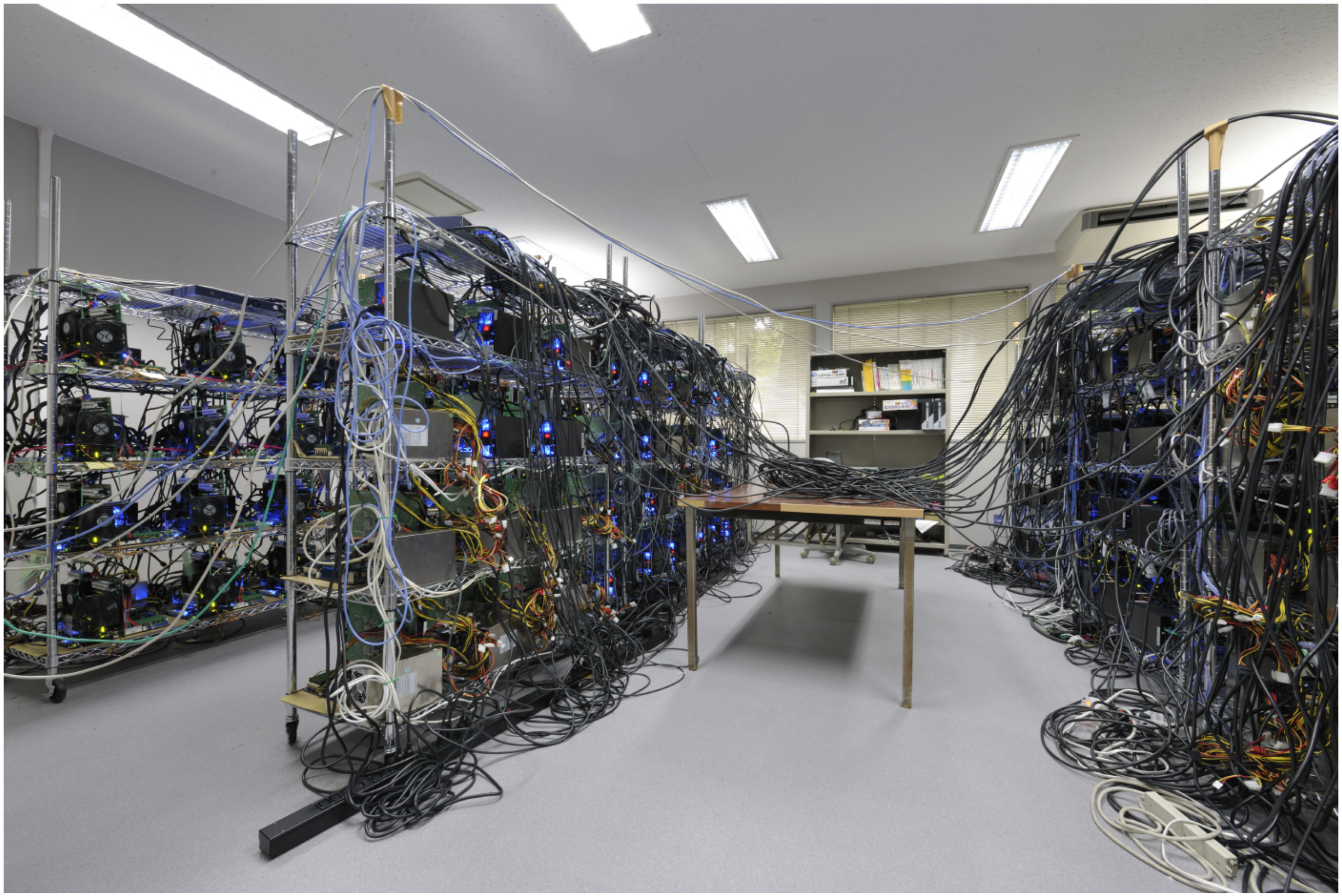
Processor board



PCIe x16 (Gen 1) interface
Altera Arria GX as DRAM controller/communication interface

- **Around 250W power consumption**
- **Not quite running at 500MHz yet... (FPGA design not optimized yet)**
- **900Gflops DP peak (450MHz clock)**
- **Available from K&F Computing Research**

GRAPE-DR cluster system



GRAPE-DR cluster system

- **128-node, 128-card system (105TF theoretical peak @ 400MHz)**
- **Linpack measured: 21 Tflops@330MHz (still lots of tunings necessary....)**
- **Gravity code: measured 1.06Tflops/board, working**
- **Host computer: Intel Core i7+X58 chipset, 12GB memory**
- **network: x4 DDR Infiniband**

New high-order integration scheme

Nitadori and JM 2008

- fourth-order Hermite scheme is now widely used.
- For many problems, higher order schemes can be advantageous.
- GRAPE-DR (unlike previous GRAPEs) can be used with whatever schemes.

Two different ways to achieve higher orders

- Use previous timesteps
- Calculate 2nd (for 6th) and 3rd (for 8th) time derivatives directly.

The latter approach

- is easier to program.
- has much smaller error coefficient
- can be made time-symmetric

Predictor and corrector

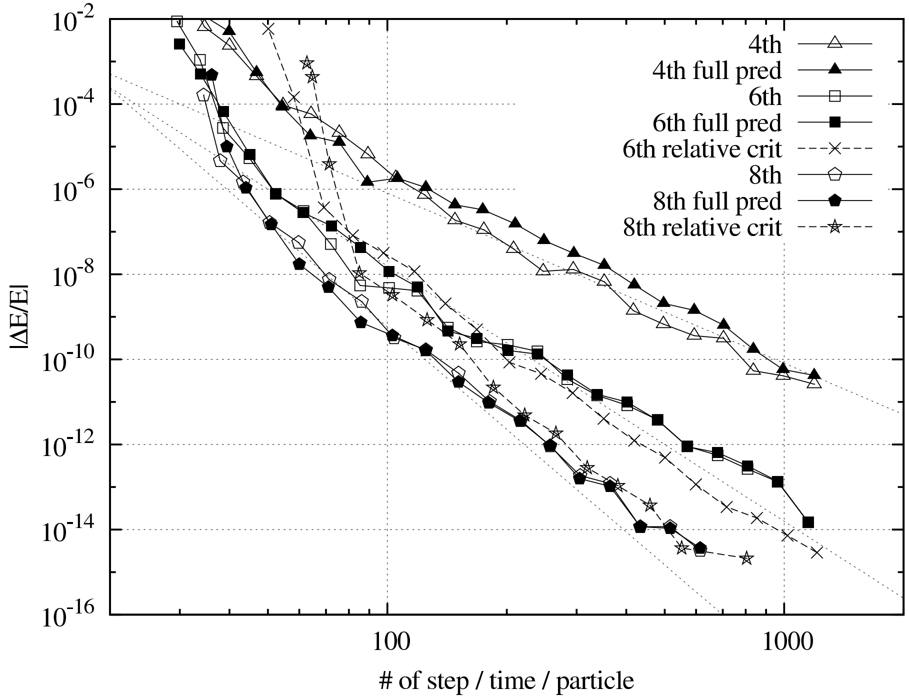
Predictors: Usual polynomial form.
Caution: need to predict acceleration (and jerk for 8th order) and need to use one previous value(s) to construct higher-order terms.

Correctors:

$$v_{i,c} = v_{i,0} + \frac{\Delta t}{2}(a_{i,1} + a_{i,0}) - \frac{\Delta t^2}{10}(j_{i,1} - j_{i,0}) + \frac{\Delta t^3}{120}(s_{i,1} + s$$

$$v_{i,c} = v_{i,0} + \frac{\Delta t}{2}(a_{i,1} + a_{i,0}) - \frac{3\Delta t^2}{28}(j_{i,1} - j_{i,0})$$
$$+ \frac{\Delta t^3}{84}(s_{i,1} + s_{i,0}) - \frac{\Delta t^4}{1680}(c_{i,1} - c_{i,0}) + O(\Delta t^9),$$

Numerical result



- $N = 1024$, Plummer model, $\epsilon = 4/N$
- Higher order schemes actually work.
- They allow much larger timesteps than that for the 4th order scheme for practical range of accuracy.

Tree-direct hybrid

(Fujii et al 2006, Oshino et al in prep.)
Basic idea: Similar to MVS, divide the potential into two terms.

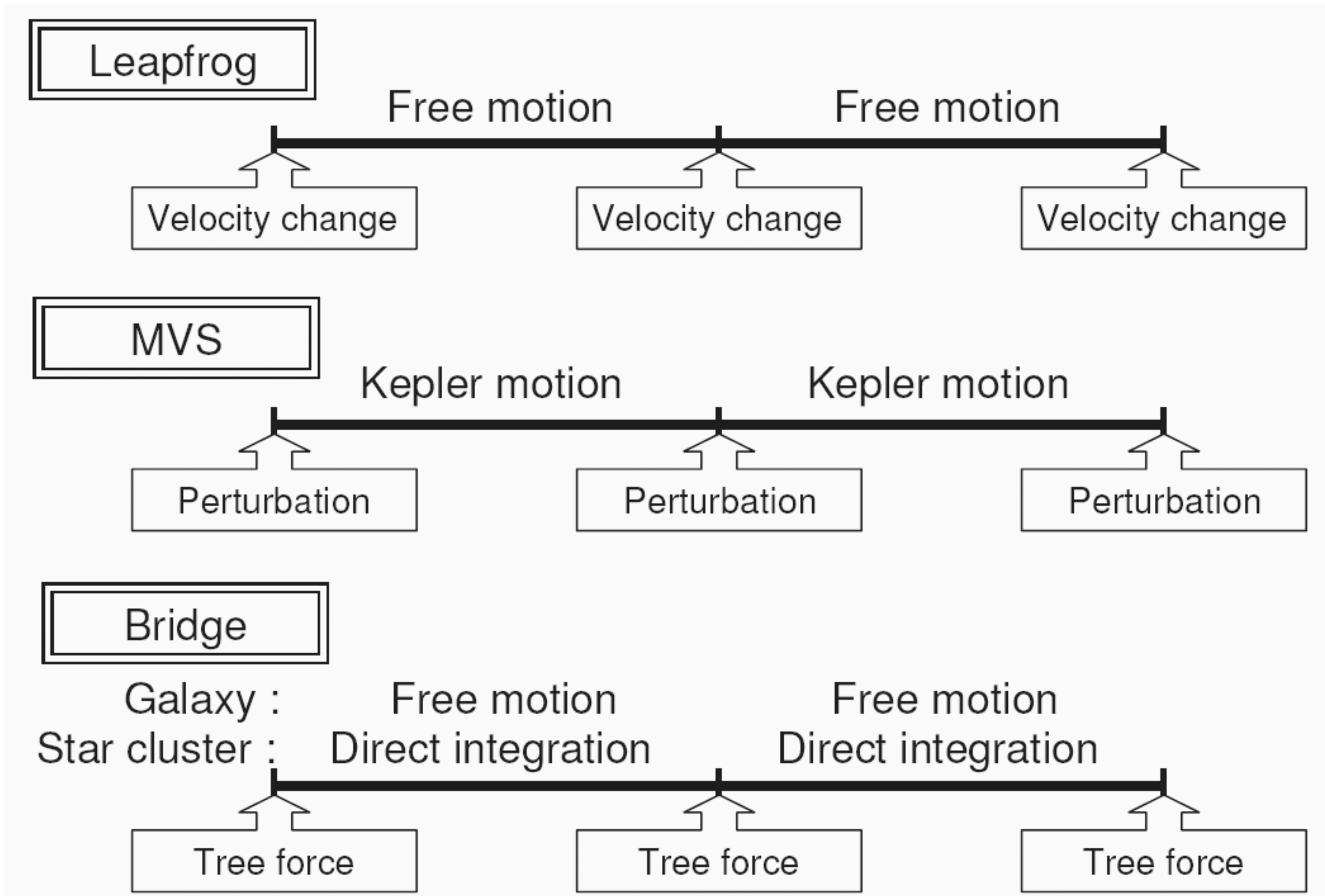
MVS: H divided into Kepler motion around the Sun and planet-planet interaction

Our method: H divided to fast-varying potential and slow-varying one.
fast varying term: Hermite with individual timestep
slow part: Leapfrog

Fujii et al: star cluster embedded in a galaxy:
Internal potential of star cluster + everything else

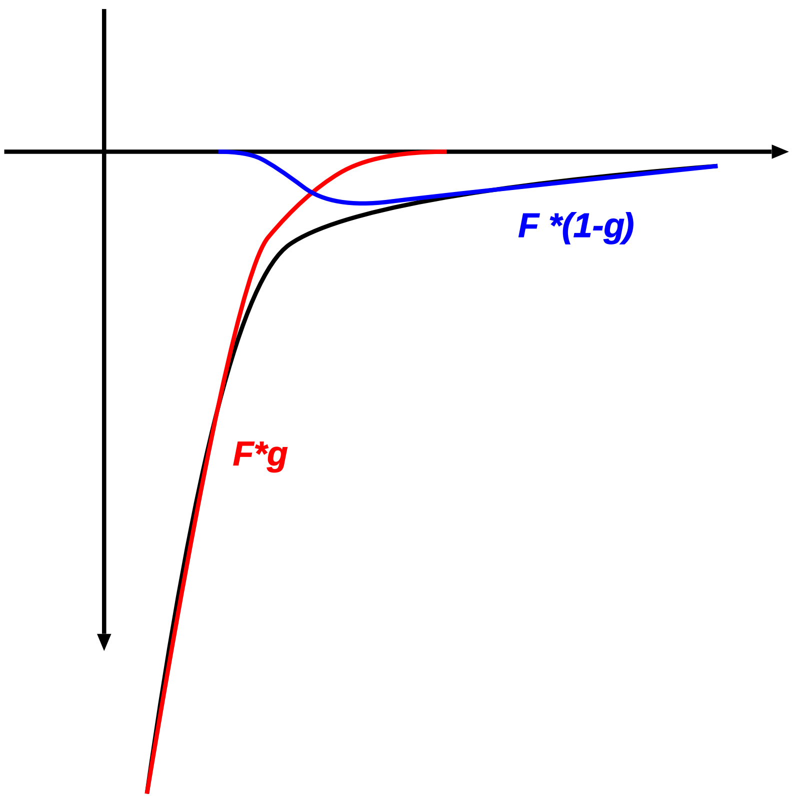
Oshino et al.: General N-body systems
Pairwise potential divided to near- and far- terms.

How does it work?



Oshino et al.

$$F_{ij} = -Gm_i m_j \frac{r_{ij}}{|r_{ij}|^3} = F_{ij}(1 - g(|r_{ij}|) + F_{ij} g(|r_{ij}|)$$



- $F * g +$ kinetic energy: individual timestep+4th (or higher) order Hermite
- $F * (1 - g)$: tree + leapfrog
- g should have compact support, and should be differentiable for appropriate number of times.

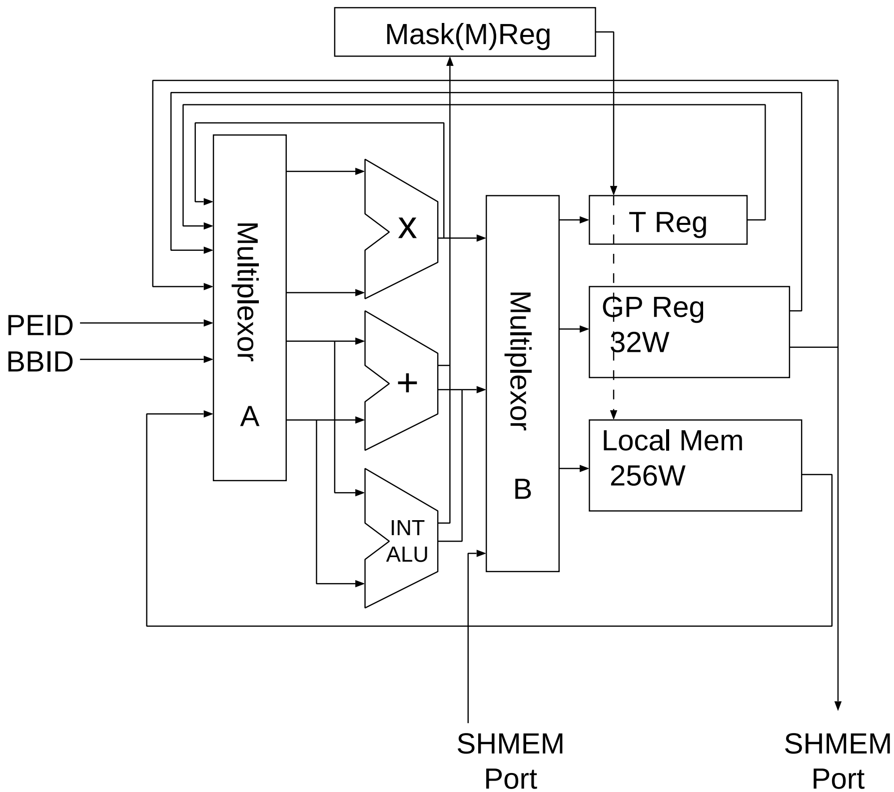
Development status

- **Original design: for planet formation**
- **Seems to work fine**
  - **Very fast: full collisional simulation with treecode speed.**
  - **Accuracy: should be enough for thermal evolution**
- **MPI Parallel version (based on parallel treecode) under development**

Summary

- **Equal-mass, circular SMBH binary would not merge through stellar dynamical effect + GW alone.**
- **Unequal-mass SMBH binary can reach high eccentricity, and merge through GW**
- **Other effects (triaxiality, gas, massive perturbers) might help**
- **Structure of parent galaxy:** $\rho \propto r^{-1/2}$

Processor architecture



- Float Mult
- Float add/sub
- Integer ALU
- 32-word registers
- 256-word memory
- communication port

Comparison with GPGPU

- **Significantly better silicon usage**
- **Higher cost per silicon area... (small production quantity)**
- **We'll see....**

How do you use it?

- GRAPE: The necessary software is now ready. Essentially the same as GRAPE-6.
- Matrix etc ... RIKEN/NAOJ will do something
- New applications:
  - Primitive Compiler available
  - For high performance, you need to write the kernel code in assembly language

Primitive compiler

(Nakasato 2006)

```
/VARI  xi, yi, zi, e2;
/VARJ  xj, yj, zj, mj;
/VARF  fx, fy, fz;
dx = xi - xj;
dy = yi - yj;
dz = zi - zj;
r2 = dx*dx + dy*dy + dz*dz + e2;
r3i= powm32(r2);
ff = mj*r3i;
fx += ff*dx;
fy += ff*dy;
fz += ff*dz;
```

- **Assembly code**
- **Interface/driver functions**

are generated from this "high-level description".

Interface functions

```
struct SING_hlt_struct0{
  double xi;
  double yi;
  double zi;
  double e2;
};
int SING_send_i_particle(struct SING_hlt_struct0 *ip,
                         int n);
...

int SING_send_elt_data0(struct SING_elt_struct0 *ip,
                        int index_in_EM);
...
int SING_get_result(struct SING_result_struct *rp);

int SING_grape_run(int n);
```