Evolution of Multiple Blackhole Systems in Galactic Centers

Jun Makino
Center for Computational Astrophysics
and
Division Theoretical Astronomy
National Astronomical Observatory of Japan
Masaki Iwasawa
University of Tokyo

Talk structure

1. SMBH-SMBH binary
   - Summary of recent results
2. SMBH triple
   - Why we consider SMBH triple?
   - Simulation result
   - Implications
3. GRAPE-DR updates
4. A few words on numerical methods
   - 6th and 8th order Hermite scheme (Nitadori)
   - Tree-direct hybrid (Fujii & Iwasawa)

SMBH-SMBH binary

- **Formed by merger of two galaxies with SMBHs**
- **"Hardens" in the way same as usual binaries in globular clusters**
- **Difference from usual binaries:**
  - **Loss-cone depletion can prevent the hardening. (e.g., Begelman, Blandford and Rees 1980)**
  - **In this regime, the hardening rate should be proportional to (relaxation time)$^{-1}$**

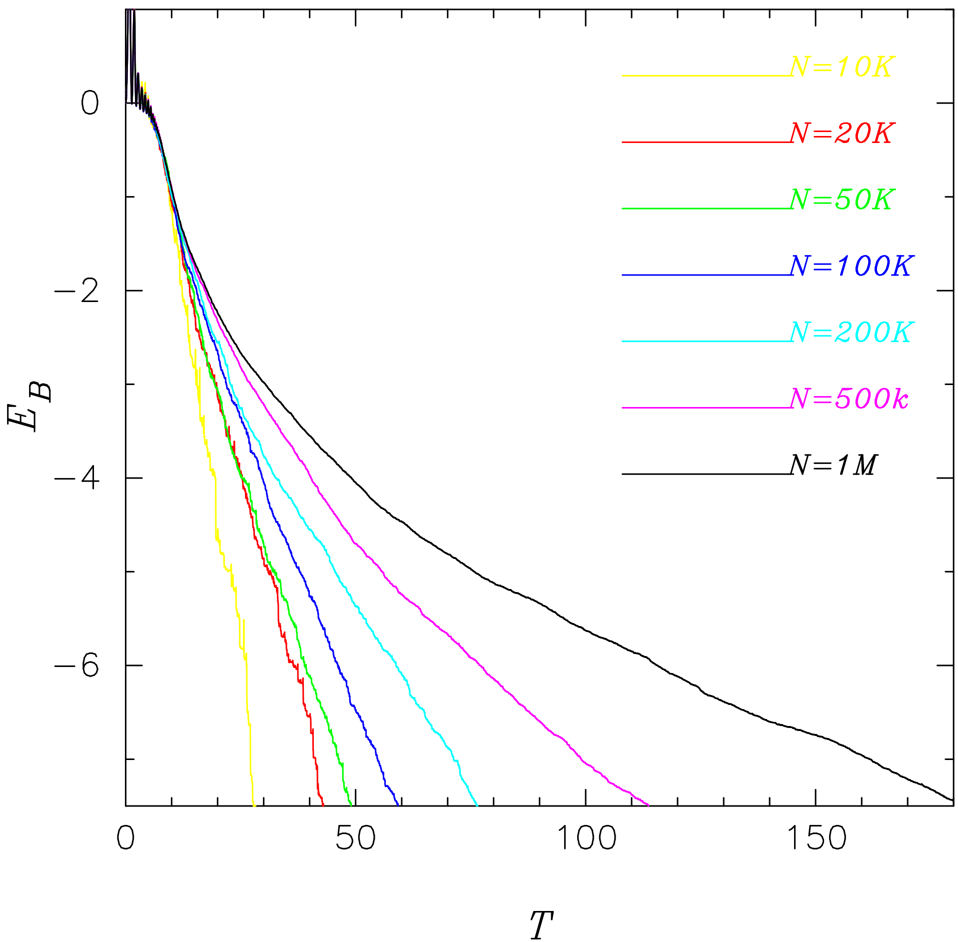
Recent N-body simulations of SMBH binaries

A few years ago:
"numerical N-body experiments are not well suited to probe these mechanisms over long times due to spurious relaxation."
(Milosavljvić and Merritt 2003)

Current situation is somewhat better:

- **JM and Funato 2004**
- **Berczik, Merritt, and Spurzem 2005**

JM and Funato 2004

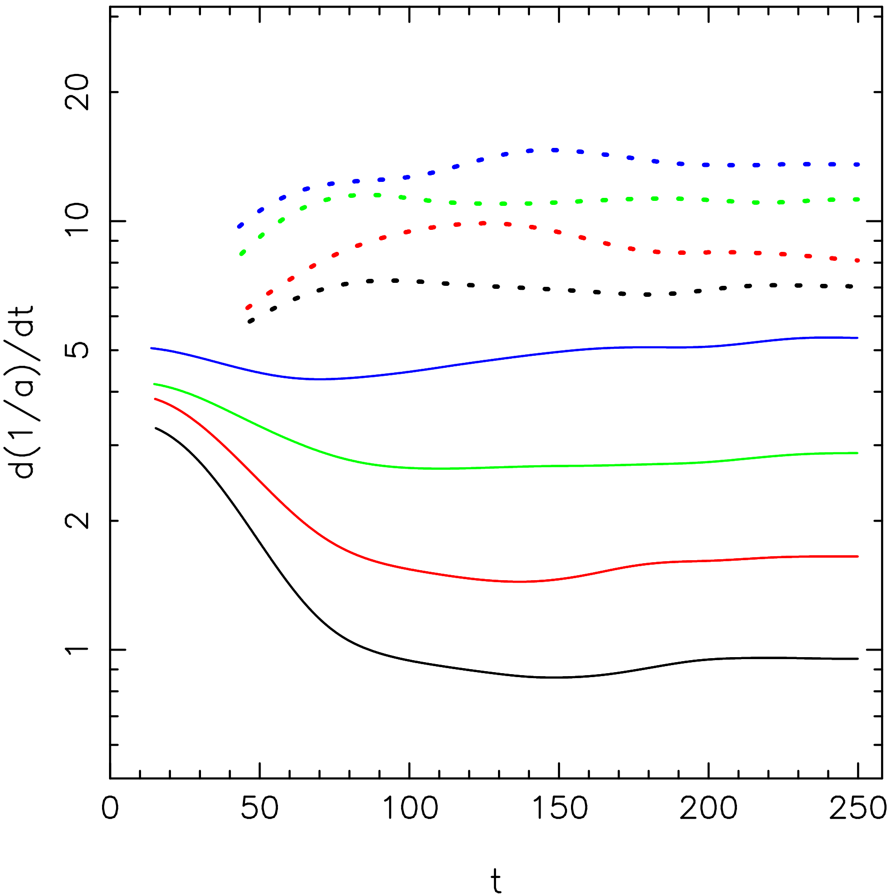


N up to 1M.
Hardening rate β depends on N.

If we write $\beta \propto N^{-\gamma}$, γ approaching to 1 for late phase

Not inconsistent with asymptotic value being 1.

Berczik et al 2005



N up to 0.4M
Simulation significantly longer than JM and F 2004.

N dependence
$\sim N^{0.8}$ ($M_{bh} = 0.02$)
$\sim N^{0.33}$??
($M_{bh} = 0.005$)

Summary of BHB N-body simulations

- N much larger than old simulations
- Duration also longer
- Growth rate shows clear dependence on N
- Results not converged yet...
- SMBH binaries do not merge in Hubble time.

Remaining problems

- Gas
- Non-spherical galaxies (Berczik et al. 2006)
- Non-equal-mass BHs (Matsubayashi et al. 2007)
- Triples (Iwasawa et al. 2006)

SMBH triples

- **If binary BHs really do not merge, merging of a galaxy hosting a binary BH and another galaxy with single BH results in SMBH triple.**
- **Quadruple BH can form if both galaxies contain binary BHs.**
- **Through triple interaction, one or more BHs might be ejected out of the parent galaxy. (Saslaw et al. 1974)**
- **Or, two of the three BHs might merge through gravitational wave radiation if the eccentricity becomes sufficiently high.**

JM and Ebisuzaki 1994

- **5-10 triple interactions are necessary before ejection.**
- **Eccentricity of the binary y after triple interaction would follow "thermal" distribution ($f(e) = 2e$)**
- **Highest eccentricity of the binary can exceed 0.95. Merging timescale can be pretty short.**

N-body simulation

- For whatever reason, not much published result for SMBH triples
- Our work (Iwasawa et al. 2006) might be the first one.

Simulation method

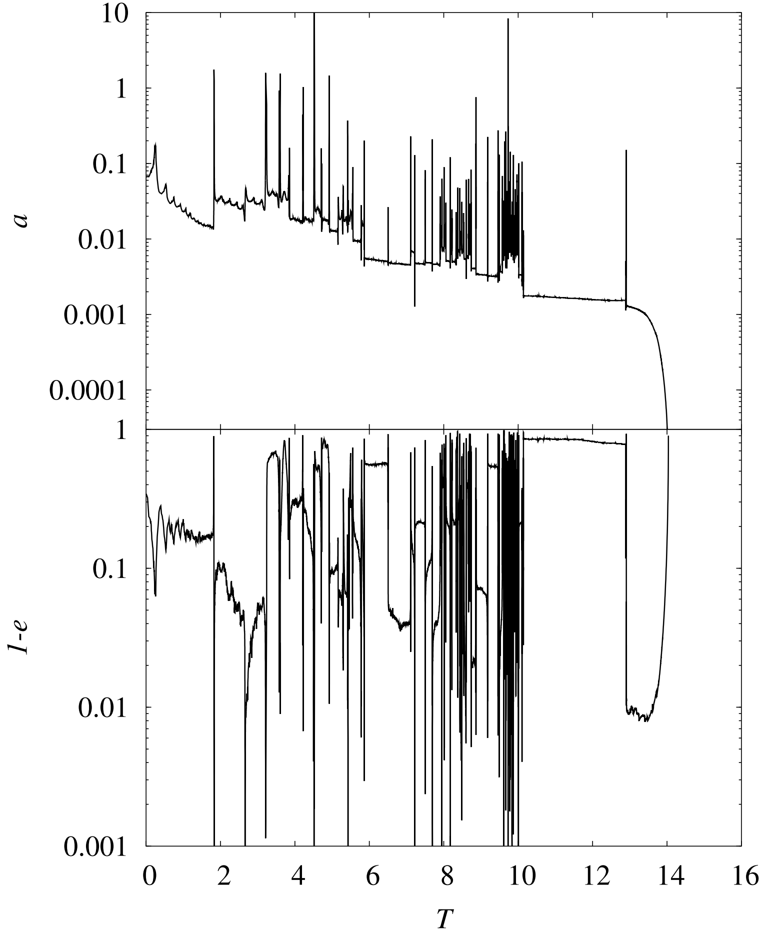
- **Direct simulation with GRAPE-6**
- **GW effect for BH-BH interaction**
- **Gravity from BH (both to BHs and stars) calculated on host**
- **No softening for BH-BH interaction**

Model parameters

- King model galaxy, $M_g = 10^{10} M_\odot$, mostly $W_0 = 7$, up to 128k particles.
- $M_{BH} = 10^8 M_\odot$
- Velocity dispersion of parent galaxy $\sigma = 300$km/s
- Two BHs: initial distance 0.1, eccentric orbit. Third one: 0.5 from the center (in Heggie units)
- Changed the initial velocity of the third one in various ways (freefall, coplanar, others)

Freefall case

a (top) and $1 - e$ (bottom)

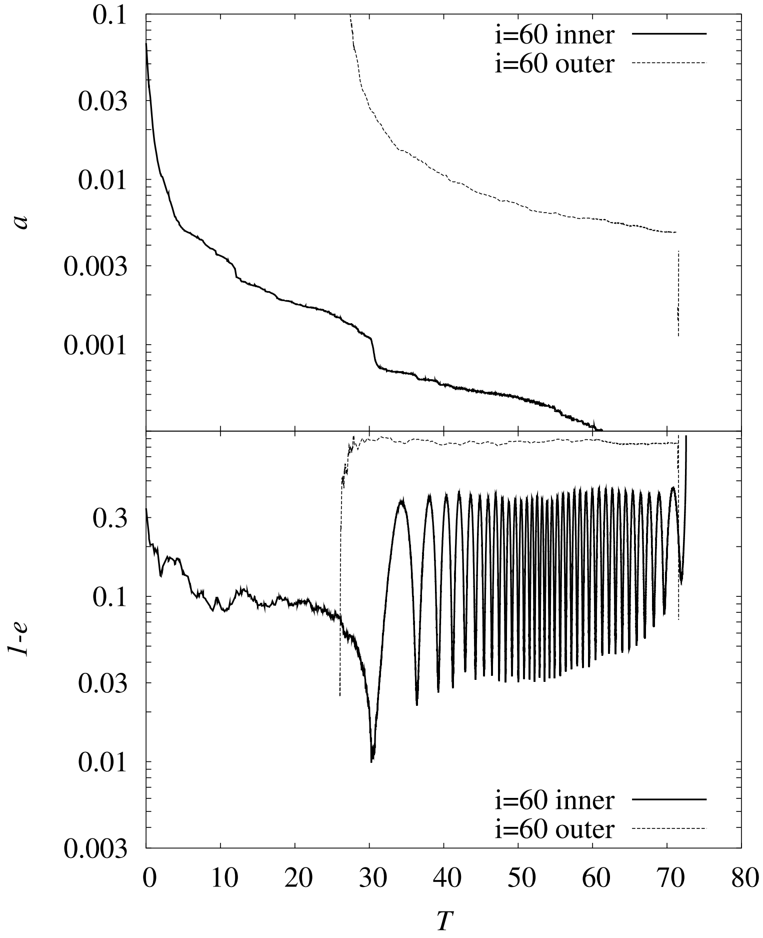


Eccentricity shows random changes after triple interaction.

In many cases, $e > 0.9$. Merged by GW radiation after e reached 0.99.

The outcome (ejection or merging) depends critically on the depth of the potential of the parent galaxy.

60 degree initial inclination



No triple interaction

Eccentricity of the inner binary oscillates through Kozai mechanism.

Finally merges through GW radiation.

Similar results for smaller inclinations.

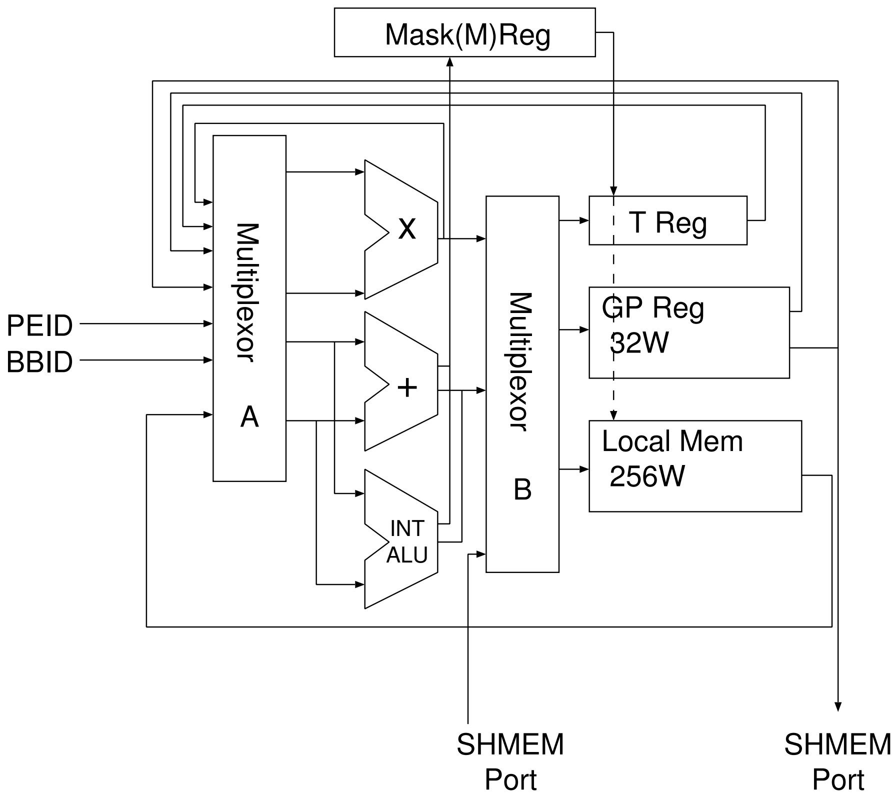
Result Summary

- **If the total angular momentum of the BH triple is small, complex three-body interactions result in either ejection or merging.**
- **If not, a hierarchical tripe forms.**
  - **If inner binary has large inclination, Kozai mechanism drives the oscillation of eccentricity, and inner binary merges.**
  - **If not, outer binary gradually shrinks, until the triple becomes unstable.**

Next-Generation GRAPE — GRAPE-DR

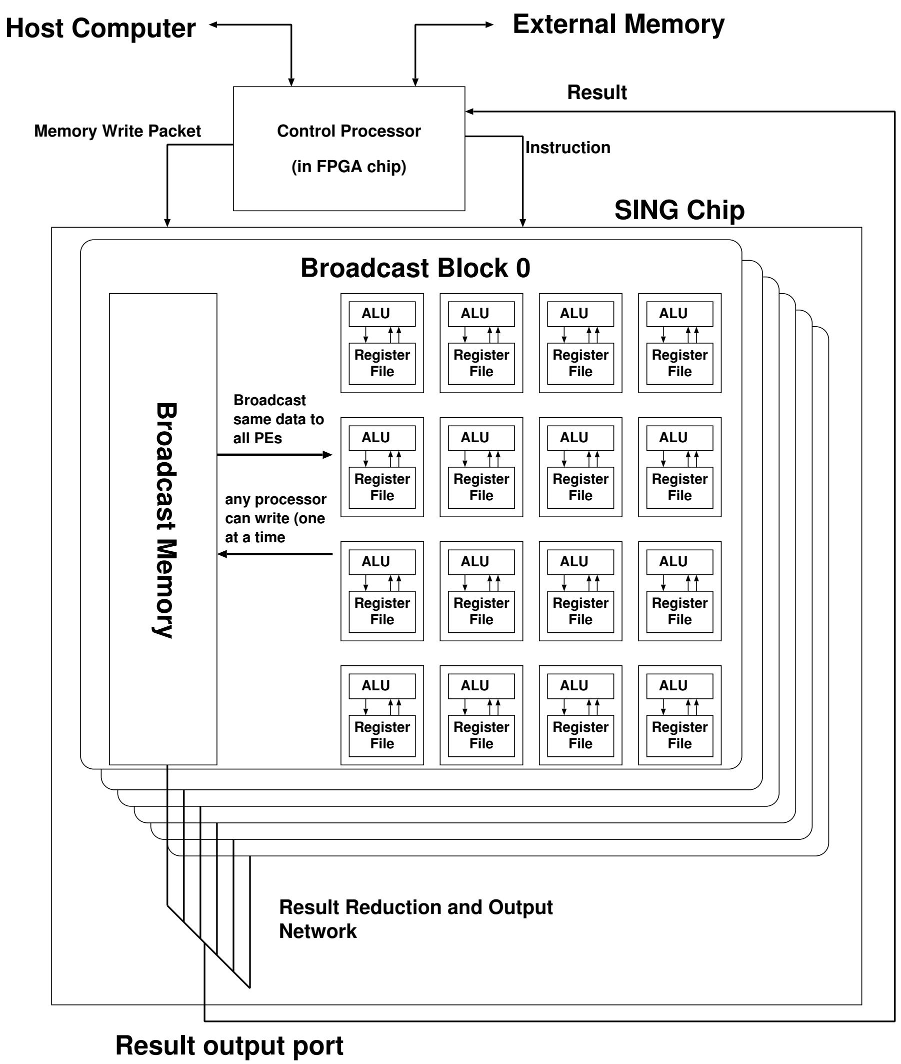
- Planned peak speed: 2 Pflops
- New architecture — wider application range than previous GRAPEs
- primarily to get funded
- No force pipeline. SIMD programmable processor
- Planned completion year: FY 2008 (early 2009)

Processor architecture



- Float Mult (24 bit mantissa, with full 49 bit output)
- Float add/sub (60 bit mantissa)
- Integer ALU (72 bit)
- 32-word (72 bit) general-purpose register file
- 256-word (72 bit) memory
- ports to shared memory (shared by 32 processors)

Chip structure



Collection of small processors.

One chip integrates 512 processors.
Single processor runs at 500MHz clock (2 operations/cycle).

Peak speed of one chip: 0.5 Tflops (20 times faster than GRAPE-6).

Why we changed the architecture?

- **To get budget (*N*-body problem is too narrow...)**
- **To allow a wider range of applications**
  - **Molecular Dynamics**
  - **Boundary Element method**
  - **Dense matrix computation**
  - **SPH**
- **To allow a wider range of algorithms**
  - **FMM**
  - **Ahmad-Cohen**
  - **...**

Comparison with FPGA

- **much better silicon usage (ALUs in custom circuit, no programmable switching network)**
- **(possibly) higher clock speed (no programmable switching network on chip)**
- **easier to program (no VHDL necessary; assembly language and compiler instead)**

Comparison with GPGPU

- **Significantly better silicon usage**
- **Higher cost per silicon area... (small production quantity)**
- **We'll see....**

How do you use it?

- **GRAPE: The necessary software is now ready. Essentially the same as GRAPE-6.**
- **Matrix etc ... RIKEN/NAOJ will do something**
- **New applications:**
  - **Primitive Compiler available**
  - **For high performance, you need to write the kernel code in assembly language**

Primitive compiler

(Nakasato 2006)

```
/VARI  xi, yi, zi, e2;
/VARJ  xj, yj, zj, mj;
/VARF  fx, fy, fz;
dx = xi - xj;
dy = yi - yj;
dz = zi - zj;
r2 = dx*dx + dy*dy + dz*dz + e2;
r3i= powm32(r2);
ff = mj*r3i;
fx += ff*dx;
fy += ff*dy;
fz += ff*dz;
```

- **Assembly code**
- **Interface/driver functions**

are generated from this "high-level description".

Interface functions

```
struct SING_hlt_struct0{
  double xi;
  double yi;
  double zi;
  double e2;
};
int SING_send_i_particle(struct SING_hlt_struct0 *ip,
                         int n);
...

int SING_send_elt_data0(struct SING_elt_struct0 *ip,
                        int index_in_EM);
...
int SING_get_result(struct SING_result_struct *rp);

int SING_grape_run(int n);
```

Development status

2nd prototype board. (Designed by Toshi Fukushige)
Difference from the 1st one:
PCI-Express x8 interface
On-board DRAM
Designed to run real applications

Summary

- **GRAPE-DR, with programmable processors, will have wider application range than traditional GRAPEs.**
- **Assembly language defined.**
- **Primitive compiler is ready.**
- **Second prototype (close to production version) is just arrived.**
- **Commercial version should be ready by... sometime this year.**

6th and 8th-order Hermite schemes

- **fourth-order Hermite scheme is not widely used.**
- **For many problems, higher order schemes can be advantageous.**
- **GRAPE-DR (unlike previous GRAPEs) can be used with whatever schemes.**

Two different ways to achieve higher orders

- Use previous timesteps
- Calculate 2nd (for 6th) and 3rd (for 8th) time derivatives directly.

The latter approach

- is easier to program.
- has much smller error coefficient
- can be made time-symmetric

Acceleration and derivatives

$$\begin{aligned}
\boldsymbol{a}_{ij} &= m_j \frac{\boldsymbol{r}_{ij}}{r_{ij}^3}, \\
\boldsymbol{j}_{ij} &= m_j \frac{\boldsymbol{v}_{ij}}{r_{ij}^3} - 3\alpha \boldsymbol{a}_{ij}, \\
\boldsymbol{s}_{ij} &= m_j \frac{\boldsymbol{a}_j - \boldsymbol{a}_i}{r_{ij}^3} - 6\alpha \boldsymbol{j}_{ij} - 3\beta \boldsymbol{a}_{ij}, \\
\boldsymbol{c}_{ij} &= m_j \frac{\boldsymbol{j}_j - \boldsymbol{j}_i}{r_{ij}^3} - 9\alpha \boldsymbol{s}_{ij} - 9\beta \boldsymbol{j}_{ij} - 3\gamma \boldsymbol{a}_{ij}.
\end{aligned}$$

Acceleration and derivatives (cont'd)

$$\alpha = \frac{\mathbf{r}_{ij} \cdot \mathbf{v}_{ij}}{r_{ij}^2},$$

$$\beta = \frac{|\mathbf{v}_{ij}|^2 + \mathbf{r}_{ij} \cdot (\mathbf{a}_j - \mathbf{a}_i)}{r_{ij}^2} + \alpha^2,$$

$$\gamma = \frac{3\mathbf{v}_{ij} \cdot (\mathbf{a}_j - \mathbf{a}_i) + \mathrm{r}_{ij} \cdot (\mathbf{j}_j - \mathbf{j}_i)}{r_{ij}^2} + \alpha(3\beta - 4\alpha^2),$$

Predictor and corrector

Predictors: Usual polynomial form.
Caution: need to predict acceleration (and jerk for 8th order) and need to use one previous value(s) to construct higher-order terms.

Correctors:

$$v_{i,c} = v_{i,0} + \frac{\Delta t}{2}(a_{i,1} + a_{i,0}) - \frac{\Delta t^2}{10}(j_{i,1} - j_{i,0}) + \frac{\Delta t^3}{120}(s_{i,1} +$$

$$v_{i,c} = v_{i,0} + \frac{\Delta t}{2}(a_{i,1} + a_{i,0}) - \frac{3\Delta t^2}{28}(j_{i,1} - j_{i,0}) + \frac{\Delta t^3}{84}(s_{i,1} + s_{i,0}) + \frac{\Delta t^4}{1680}(c_{i,1} - c_{i,0}) + O(\Delta t^9),$$

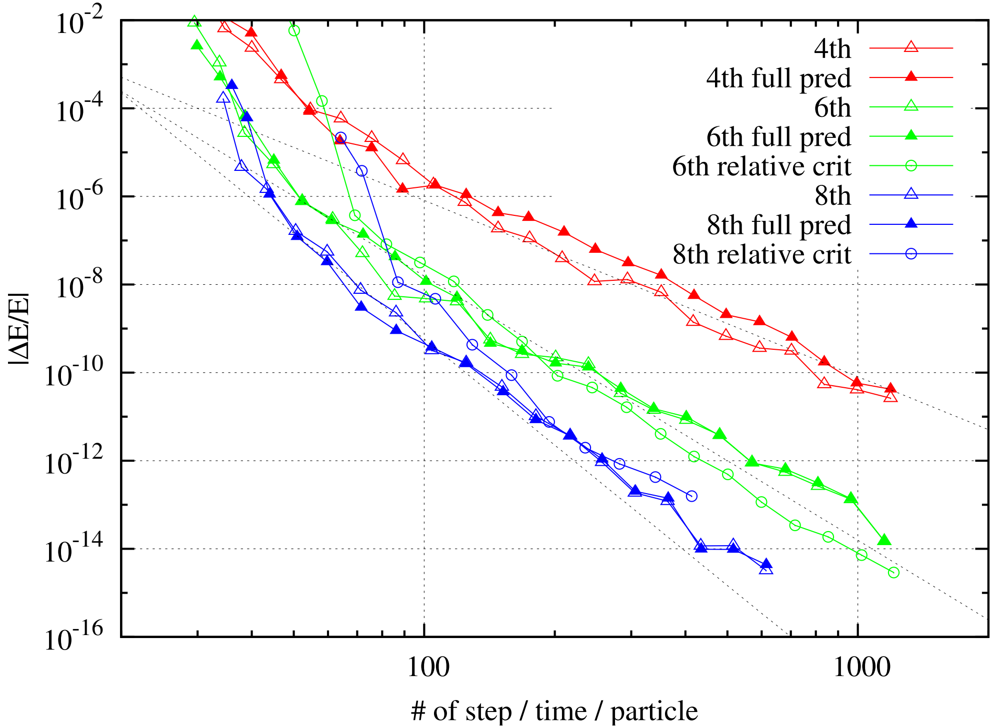
Timestep criterion

"Generalization" of the standard one:

$$\Delta t = \eta \left(\frac{|\boldsymbol{a}^{(0)}||\boldsymbol{a}^{(2)}| + |\boldsymbol{a}^{(1)}|^2}{|\boldsymbol{a}^{(p-3)}||\boldsymbol{a}^{(p-1)}| + |\boldsymbol{a}^{(p-2)}|^2} \right)^{1/(2p-6)} .$$

seems to work fine.

Numerical result



- $N = 1024$, Plummer model, $\epsilon = 4/N$
- Higher order schemes actually work.
- They allow much larger timesteps than that for the 4th order scheme for practical range of accuracy.

Tree-direct hybrid

Evolution of compact star clusters near the galactic center

- **thermal evolution**
- **dynamical friction**
- **tidal disruption**
- **stellar evolution**

all proceed in similar timescales.
To follow orbital evolution accurately, parent galaxy should be modeled as an N-body system.
Practical problem: calculation cost would be too high if direct method is used for entire system.

Need for a hybrid

- **Parent galaxy requires fast algorithm**
- **star clusters require accurate algorithm**

Our approach

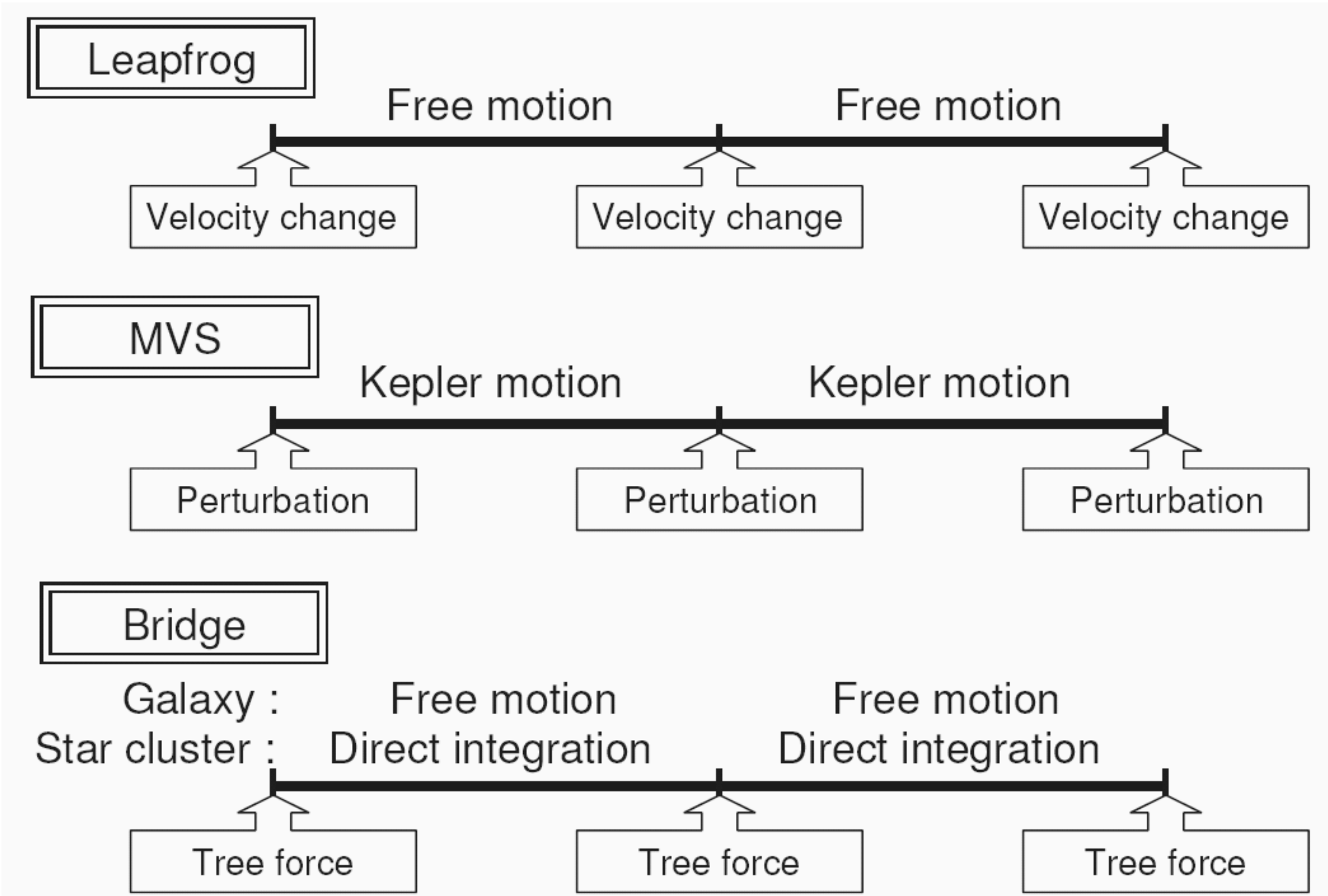
Similar to MVS.

MVS: divide Hamiltonian to Kepler motion of planets and planet-planet interaction.

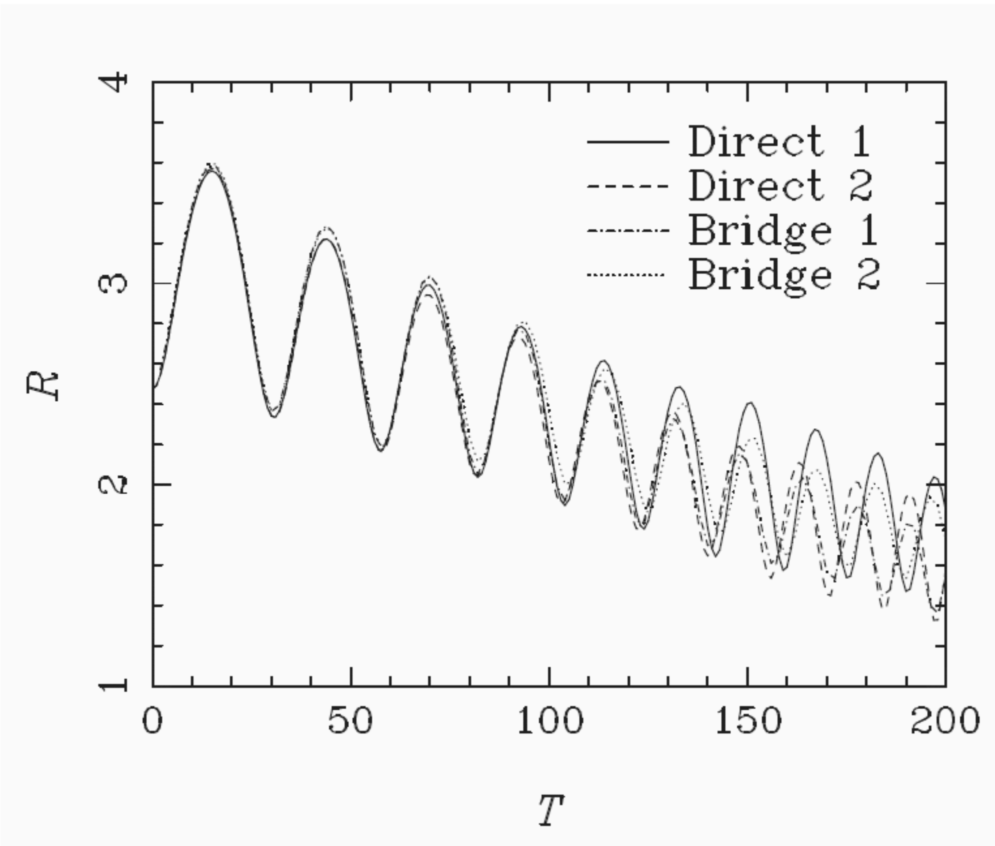
Our scheme: divide Hamiltonian to (Potential energy except internal potential of cluster) and (kinetic energy plus cluster potential)

BRIDGE (Bridge is for Realistic Interactions in Dense Galactic Environment)

How does it work?



Test result



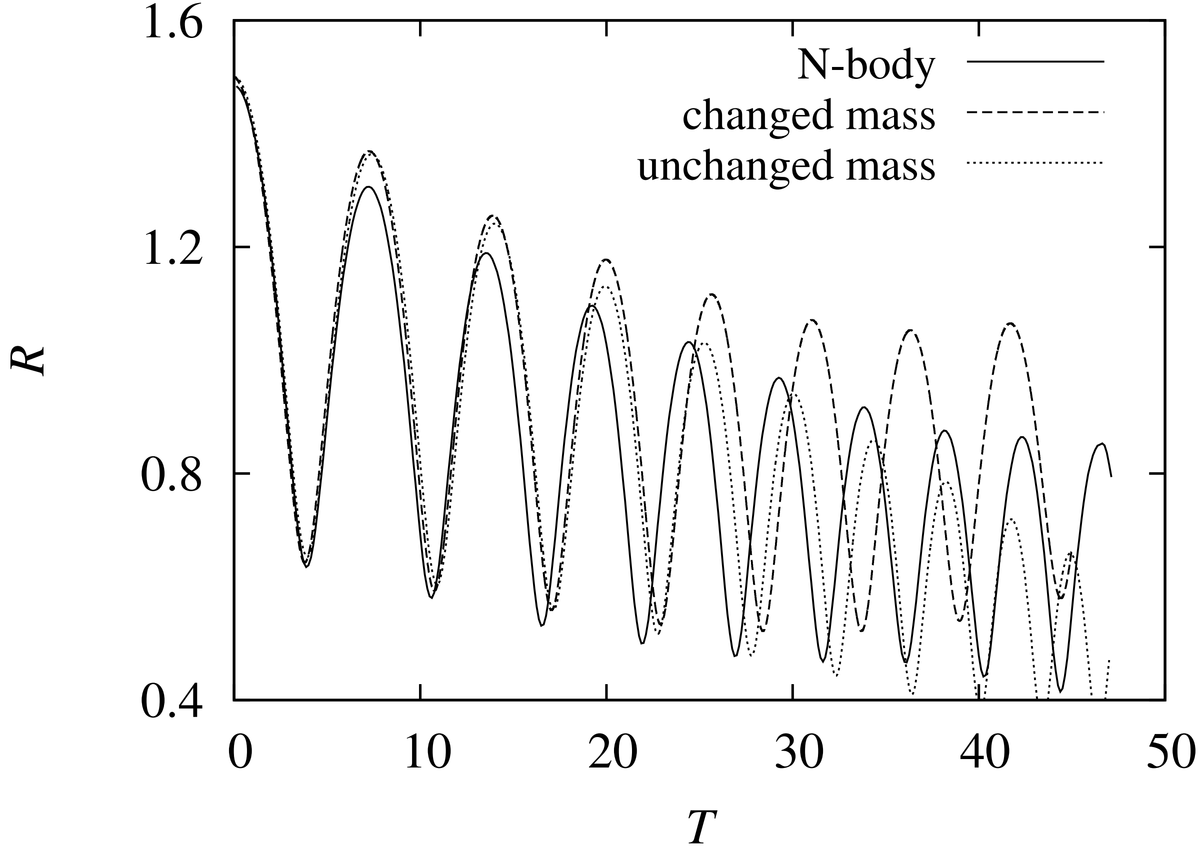
- $N = 100\text{k} + 2\text{k}$
- Similar model as in Fujii et al. 2996
- Two runs: different random seeds
- Results agree well.
- Energy error: dominated by the parent galaxy.

Summary

- New hybrid method combines direct and tree
- Based on the idea similar to MVS
- Fairly simple to implement.
- Fast (for the parent galaxy) and accurate (for star clusters)
- In principle, existing direct codes (kira or NBODYx) can be embedded into treecode.

Orbital evolution of cluster with DF

Fujii et al. 2006: Satellite galaxy N-body simulation



In full-nbody simulation, satellite falls faster.

Why?

- **Satellite gives angular momentum to escaped stars**

- **escaped stars, while remaining close to the satellite, enhance the dynamical friction**